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2.1. Let U, V and W be subspaces of an inner product space X, and suppose that $U \perp V$, $U \perp W$, and that $U + V = U + W$. Prove that $V = W$.

2.2. Prove that for any $z \in l^2(\mathbf{Z}_N)$, we have $\widehat{U(z)}(n) = \hat{z}(n)$ for all n, and that, if N is even, $\widehat{D(z)}(n) = \frac{1}{2}(\hat{z}(n) + \hat{z}(n + \frac{1}{2}N))$ for all n. Here U and D denote upsampling and downsampling, respectively.

2.3. Let $u \in l^2(\mathbf{Z}_N)$ and define $v(n) = u(n) + u(n + \frac{1}{2}N)$, assuming N even. Prove that $\hat{v}(n) = \hat{u}(2n)$ for all n.

2.4. Define $z = (1, 2, 3, 4, 5, 6, 7, 8) \in l^2(\mathbf{Z}_8)$. Calculate $P_{-j}(z)$ and $Q_{-j}(z)$, $j = 1, 2, 3$, in the Haar wavelet system. Notation as in Frazier.

2.5. For any $z \in l^\infty(\mathbf{Z})$ let us denote by $K(z)$ the smallest interval $I \subset \mathbf{Z}$ such that $z(j) = 0$ for all $j \notin I$. ($K(0) = \emptyset$.) Prove that if $K(z)$ and $K(w)$ are of finite length, then $K(z * w) = K(z) + K(w)$. Show by an example that this may not be true if $K(z)$ or $K(w)$ is infinitely long. But it is true if $K(z)$ and $K(w)$ are contained in the set of nonnegative integers.

2.6. Prove that the convolution equation $a * z = \delta$ cannot be solved with $z \in l^2(\mathbf{Z})$ if $a(0) = a(1) = 1$ and $a(k) = 0$ for $k \neq 0, 1$. Here δ is the Dirac δ, the vector with 1 at position zero and 0 at all other slots. Are there solutions in some other space $l^p(\mathbf{Z})$?

2.7. Prove that $\hat{z}$ is a continuous function, $\hat{z} \in C(\mathbf{R})$, if $z \in l^1(\mathbf{Z})$. Here $\hat{z}(t) = \sum z(k)e^{ikt}$, $t \in \mathbf{R}$.

2.8. Prove that if $f \in C^2(\mathbf{R})$ is periodic with period 2π, then $\check{f} \in l^1(\mathbf{Z})$. Here $\check{f}(n) = (2\pi)^{-1} \int_{-\pi}^{\pi} f(t)e^{-int}dt$, $t \in \mathbf{R}$.

2.9. Prove that the convolution equation $a * z = b$ can be solved with $z \in l^1(\mathbf{Z})$ if $\hat{a}, \hat{b} \in C^2(\mathbf{R})$ and have period 2π, and if in addition we assume that there exists an $\varepsilon > 0$ such that $|\hat{a}(t)| \geqslant \varepsilon$ for all t such that $\hat{b}(t) \neq 0$.

2.10. Prove that the convolution equation $z * z = \delta$ has exactly two solutions $z \in l^1(\mathbf{Z})$, but infinitely many solutions $z \in l^2(\mathbf{Z})$.