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ÖVNINGAR, Blad 1
Transformer för beräkningar
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1.1. Calculate $(1+i\sqrt{3})^6$ (a) using polar representation; (b) using the binomial formula... if you do not get too tired, that is.

1.2. Solve completely the equation $z^7 = 7$, $z \in \mathbf{C}$.

1.3. Find the dimension of the space of all polynomials of one complex variable which are of degree at most 17 and vanish at the points $z = 0$, $z = 1$, $z = 2$.

1.4. Is the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ diagonizable?

1.5. Let V be the space of all polynomials of one real variable of degree at most m. Is the mapping $T\colon V \ni f \mapsto f' \in V$ injective? Is it surjective? Determine the dimensions of $\operatorname{im} T$ and $\ker T$. Determine the eigenvalues of T as well as all eigenvectors. Is T diagonizable?

1.6. We compare convolution on $\mathbf{Z}$ and on $\mathbf{Z}_N = \mathbf{Z}/N\mathbf{Z}$, $N \geqslant 2$. Let $a = \delta_{-1} - \delta_0$, where we define $\delta_k(j)$ to be zero for $j \neq k$ and to be one for $j = k$. Prove that on $\mathbf{Z}$ the equation $a * z = b$ has a solution for any given b, and that the space of solutions to $a * z = 0$ has dimension 1. By way of contrast, show that on $\mathbf{Z}_N$, the equation $a * z = b$ can be solved only if b satisfies a certain condition—and find this condition. For the homogeneous equation, the situation for $\mathbf{Z}_N$ is the same as for $\mathbf{Z}$. (The lesson is that, on $\mathbf{Z}$, the equation $a * z = b$ with periodic b can have nonperiodic solutions z.)

1.7. Define a mapping $T\colon l^2(\mathbf{Z}_4) \to l^2(\mathbf{Z}_4)$ by

$$T(z) = (2z(0) - z(1), iz(1) + 2z(2), z(1), 0).$$

Is T linear? Is T translation invariant? Calculate $T(R_1 z)$ and $R_1(Tz)$ for $z = (1, 0, -2, i)^{\mathrm{T}}$.

1.8. Let S, T be two translation-invariant linear mappings of $l^2(\mathbf{Z}_N)$ into itself. Prove that they commute, i.e., that $T \circ S = S \circ T$. Deduce that circulant matrices always commute—try also to prove this fact by a direct calculation using the definition of circulant matrices.

1.9. Prove that $T(z * w) = T(z) * w = z * T(w)$ if T is a translation-invariant linear mapping of $l^2(\mathbf{Z}_N)$ into itself.

1.10. Prove that a linear mapping T of $l^2(\mathbf{Z}_N)$ into itself is translation invariant if and only if it is a polynomial in the translation operator R_1.