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ÖVNINGAR, Blad 2
Transformer för beräkningar
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2.1. Suppose we know that $a \in l^2(\mathbf{Z}_N)$ satisfies $\hat{a}(0) = 0$, $\hat{a}(k) \neq 0$, $k = 1, ..., N-1$. Can you determine the dimension of

$$\{z \in l^2(\mathbf{Z}_N);\ a * z = 0\}?$$

Can you determine the dimension of

$$\{a * z;\ z \in l^2(\mathbf{Z}_N)\}?$$

Generalize to an arbitrary convolution equation $a * z = b$, where a, b are given vectors, z an unknown vector. What is the rule connecting the two dimensions?

2.2. Calculate $\hat{z}$ when $z = (1, 2, 3, 4)^{\mathrm{T}} \in l^2(\mathbf{Z}_4)$. Calculate $\widehat{w}$ when

$$w = (1, 2, 3, 4, 5, 6)^{\mathrm{T}} \in l^2(\mathbf{Z}_6).$$

2.3. Let $u = (1, 3)^{\mathrm{T}}$, $v = (0, 4)^{\mathrm{T}}$, $z = (1, 0, 3, 4)^{\mathrm{T}}$. Compute $\hat{u}$, $\hat{v}$ and then $\hat{z}$ using the fast Fourier transform. Compute the Fourier transform of $w = (0, 1, 4, 3)^{\mathrm{T}}$.

2.4. Compute the Fourier transform of $w = (1, 0, 1, 0, 1, 0, 1, 0)^{\mathrm{T}}$—maybe using $z = (1, 1, 1, 1)^{\mathrm{T}}$.

2.5. Let U, V and W be subspaces of an inner product space X, and suppose that $U \perp V$, $U \perp W$, and that $U + V = U + W$. Prove that $V = W$.

2.6. Prove that for any $z \in l^2(\mathbf{Z}_N)$, we have $\widehat{U(z)}(n) = \hat{z}(n)$ for all n, and that, if N is even, $\widehat{D(z)}(n) = \frac{1}{2}(\hat{z}(n) + \hat{z}(n + \frac{1}{2}N))$ for all n. Here U and D denote upsampling and downsampling, respectively.

2.7. Let $u \in l^2(\mathbf{Z}_N)$ and define $v \in l^2(\mathbf{Z}_{N/2})$ by $v(n) = u(n) + u\big(n + \frac{1}{2}N\big)$, assuming N even. Prove that $\hat{v}(n) = \hat{u}(2n)$ for all n.

2.8. Define $z = (1, 2, 3, 4, 5, 6, 7, 8) \in l^2(\mathbf{Z}_8)$. Calculate $P_{-j}(z)$ and $Q_{-j}(z)$, $j = 1, 2, 3$, in the Haar wavelet system. Notation as in Frazier.