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DAGBOK
Distributionsteori D
1999 05 19 99course

Distribution Theory D, Spring 1999. The lectures day by day.

The textbook is Hörmander's *The Analysis of Linear Partial Differential Operators, I.* The chapters to be studied are basically Chapters I–V and VII. However, certain more difficult parts can be skipped. This means that the course, somewhat tentatively, can be described as follows.

Chapter I: 1.2; 1.3 except Theorem 1.3.5 and higher; Theorems 1.4.1 and 1.4.4.

Chapter II: 2.1; 2.2; 2.3 except the proof of Theorem 2.3.6 and what follows.

Chapter III: 3.1 except Theorems 3.1.14 and 3.1.15 (Theorem 3.1.10 needs to be studied only in one variable); 3.2 and 3.3 only lightly.

Chapter IV: 4.1 except that what comes after Theorem 4.1.7 needs to be studied only lightly; 4.2; 4.3 except the proof of Theorem 4.3.3; 4.4 and 4.5 for information only.

Chapter V: For information only.

Chapter VII: 7.1 up to and including Example 7.1.17; 7.2 lightly; Theorem 7.3.1 but the rest lightly; 7.9 for information.

More exact information will be provided as we go along.

Day by day:

1. 02 03: A brief history. S. L. Sobolev and Laurent Schwartz. Different ways to define distributions. Distributions as actions on blurry points. Distributions as limits of Cauchy sequences. Distributions as derivatives of continuous functions. Constructions of test functions.

2. 02 04: Definition of distributions as linear forms satisfying an estimate on each compact set. Convolution $C^0_0 * C^0 \subset C^0$; $C^k_0 * C^0 \subset C^k$; $C^k_0 * C^p \subset C^{k+p}$. The support of a function. Locally uniform convergence of $f * \varphi$ to f as $\operatorname{supp}\varphi \to \{0\}$ if $f \in C^0$. Definition of $\partial_j u$, $u \in \mathcal{D}'(\Omega)$. The Heaviside function and its derivative, the Dirac distribution.

3. 02 11: Distribution solutions of the Laplace equation and the wave equation. Distributions defined as linear forms such that $u(\varphi_j) \to 0$ when $\varphi_j \to 0$ in $\mathcal{D}(\Omega)$. The order of a distribution. Distributions of finite order. Positive distributions are measures.

4. 02 17: Functions with $f_{xy} \neq f_{yx}$ at a point. The order of $\partial^\alpha \delta$ is $|\alpha|$. Convergence of distributions (topology $\sigma(\mathcal{D}', \mathcal{D})$, also called weak-star convergence; it agrees with the weak topology $\sigma(\mathcal{D}', \mathcal{D}'')$ since $\mathcal{D}$ is reflexive). Restrictions of distributions to a smaller open set. Partition of unity.

5. 02 18: Review of the notion of compactness. Partition of unity, cont'd. The support of a distribution. Multiplication by a smooth function. Distributions of compact support: $\mathcal{E}'(\Omega)$.

6. 02 24: The derivatives of a distribution. Trigonometric series converging in

$\mathcal{D}'(\mathbf{R})$. Distributions solving ordinary differential equations: $u' = 0$ and $u' = v$; higher order. Division of distributions by polynomials (one variable).

7. 02 25: Limits of distributions. Impossibility of estimates of distributions using restrictions of $\partial^\alpha\varphi$ to the support of the distribution. If φ is flat on $\operatorname{supp} u$ then $u(\varphi) = 0$. Boundary values of holomorphic functions of temperate growth.

8. 03 03: Fundamental solutions to the operator $(d/dx)^m$. The three convolutions $C^0(\mathbf{R}^n) * C_0^0(\mathbf{R}^n)$; $\mathcal{D}'(\mathbf{R}^n) * \mathcal{D}(\mathbf{R}^n)$, $\mathcal{E}'(\mathbf{R}^n) * \mathcal{E}(\mathbf{R}^n)$; $\mathcal{D}'(\mathbf{R}^n) * \mathcal{E}'(\mathbf{R}^n)$. Approximation of a distribution by smooth functions. Every distribution on $\mathbf{R}$ can be represented as a limit $\lim_{\varepsilon\to 0+}(f(x+i\varepsilon) - f(x-i\varepsilon))$, where f is holomorphic in $\mathbf{C} \smallsetminus \mathbf{R}$ and of temperate growth at $\mathbf{R}$. The distributions as the smallest local space which contains $C(\Omega)$ and is stable under differentiation.

9. 03 04: The index of the operators $f \mapsto f'$ and $f \mapsto xf$. The Fourier transformation on $\mathcal{S}(\mathbf{R}^n)$.

10. 03 11: The Fourier transformation is an isomorphism of $\mathcal{S}$ onto itself. The Fourier transformation on $\mathcal{S}'$.

11. 03 17: The Fourier transformation is an isometry of L^2 onto itself. The Fourier transform of a distribution with compact support can be defined pointwise.

12. 03 18: The Fourier tranform of a distribution with compact support is an entire function of exponential type. The Paley–Wiener theorem. Characterization of $\widehat{\mathcal{D}}$.

13. 03 22: Characterization of $\widehat{\mathcal{E}'}$. Solving $P(D)u = f$ with $u \in \mathcal{E}'$.

14. 03 25: Lecture by Bengt Eliasson on distributions defined as principal values of integrals. Density of exponential solutions in the space of all solutions in a convex open set.

15. 03 29: Density of exponential solutions in the space of all solutions in a convex open set, cont'd. The singular support of a distribution and estimates on its Fourier transform determining the convex hull of its singular support.

16. 04 01: Periodic distributions. Both the classical Fourier transformation and the classical Fourier series are contained in the theory of the Fourier transformation of $\mathcal{S}'$.

17. 04 07: Lecture by Henrik Brandén on fundamental solutions. Lecture by Gustaf Strandell on pseudofunctions.

18. 04 15: Lars Larsson-Cohn talar om Sobolevrum.

19. 04 21: Peter Sunehag speaks about the wave front. Hiroshi Yamagucho och Norman Levenberg listen in.

20. 04 29: Problemgenomgång.

21. 05 12: Problemgenomgång.

22. 05 19: Genomgång av några problem med Peter Sunehag och Henrik Brandén.

Notes: *Some especially interesting distributions.*

Word list: all names for basic notions in distribution theory should be learned in both Swedish and English and possibly other languages.