

Euclid's straight lines: An experience

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Contents

1. Introduction
2. The first submission
3. The second submission
4. The third submission
5. Comments
6. Sabetai Unguru
7. The fourth submission: Finally accepted

1. Introduction

In 2007—possibly earlier—I started to write a paper entitled *Euclid's straight lines*. During the academic year 2010-2011 I studied Classical Greek for Ove Strid in order to improve my understanding of Euclid (but not only).

I was aware that publication might be a problem, since the paper contained too much of Classical Greek for a mathematics journal, and too much mathematics for a language journal. I submitted it three times without success. The fourth time the paper was accepted. This note tells the story of my attempts . . . maybe somebody could learn something from my experience . . . perhaps even beyond the ever-present

Doch die Wissenschaft, man weiß es,
achtet nicht des Laienfleißes.

(Morgenstern 1952)

2. The first submission

I first submitted the paper to *Revue d'histoire des mathématiques* on 2012-04-12 and received a decision on 2012-07-12 (after three months):

Date: Thu, 12 Jul 2012 22:45:28 +0200
From: Norbert Schappacher <schappacher@math.unistra.fr>
To: kiselman@math.uu.se
Subject: RHM 241 - your submission to Revue d'histoire des mathématiques
Dear colleague :

thank you very much for having submitted your paper « Euclid's straight lines » to the Revue d'histoire des mathématiques. I have myself looked at it and two colleagues have read it in detail.

Unfortunately, your argument - although not uninteresting in itself - seems to us to clearly fall outside the scope of our journal. Even though you do allude to several historical studies, your point is basically mathematical or logical in nature and does not appear to yield conclusions of immediate interest to professional historians of mathematics.

You may want to submit this paper to a journal specializing in the philosophy of mathematics or logic.

Yours sincerely

Norbert Schappacher

3. The second submission

I next submitted the paper to *Centaurus* on 2012-07-25 and received a decision on 2012-07-31 (after 6 days):

Date: Tue, 31 Jul 2012 09:46:16 -0400 (EDT)
From: i.h.stamhuis@vu.nl
To: kiselman@math.uu.se
Subject: Centaurus - Decision on Manuscript ID CNT-OA-Jul-12-0292
31-Jul-2012

Dear Prof. Kiselman:

Thank you for submitting your manuscript # CNT-OA-Jul-12-0292 entitled "Euclid's straight lines" to *Centaurus*.

Unfortunately, we are unable to publish your article on this occasion as it does not meet the remit of the journal.

Full length articles for *Centaurus* should address issues of current interest to the European history of science community. The aim of the journal is to cover a broad range of subjects, bringing new perspectives to continuing discussions as well as introducing original research and addressing trends and techniques in the study of the history of science.

Your paper discusses a very specialized topic and not a topic of interest for a broader history of science community. It is much too technical for the readership of *Centaurus*. That it does not fit in a broad history of science journal is also clear from the references; there are no references to broader history of science literature.

However it seems to me a scholarly paper. It is possible that more specialized historical journals like 'Historia Mathematica' or 'Archive for the History of the Exact Sciences' are interested.

Although we are not able to accept your submission on this occasion, if you feel that you have other work that fits with the journal's editorial profile, please get in contact again. You may also like to visit the journal's website where you will find the most recent issues of *Centaurus*:

<http://www.blackwellpublishing.com/journal.asp?ref=0008-8994>

Once again, we thank you for your interest in *Centaurus*.

Best regards,

Prof. Ida Stamhuis

Editor in Chief, *Centaurus*

4. The third submission

I then submitted the paper to *Archive for history of exact sciences* on 2012-08-30 and received a decision on 2012-12-10 (after more than three months):

Date: Mon, 10 Dec 2012 08:59:38 +0000
 From: Jeremy Gray (j.j.gray@open.ac.uk)
 To: Christer Kiselman (kiselman@math.uu.se)
 Subject: Your paper to AHES

Dear Christer Kiselman

I have had the reply back from the editor of Archive for history of exact sciences to whom I sent your article, and I regret to say that he does not find it suitable for publication in the journal. He finds, and I agree, that the article draws on no evidence other than Euclid's Elements to illuminate how straight lines and other geometrical objects and relations were understood in the period, and that it is too much of an anachronism to compare Euclid's treatment with a modern discussion of the projective plane. I would add that the historical approach would be attempt to see how issues about lines in the plane were formulated, and then revised, as the concept of a projective plane was itself worked out in the course of the 19th century.

Yours sincerely

Jeremy Gray

—

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5. Comments

5.1. *Revue d'histoire des mathématiques*

I was surprised that it took three months to find out that my “point is basically mathematical or logical in nature” as well as by the statement that this would be a reason why “professional historians of mathematics” would not be interested.

There is an encrypted message here, I suspect. There must be a special meaning of the expression “mathematical or logical in nature” that (1) takes three months to find out rather than 45 seconds; and (2) implies that “professional historians of mathematics” are not interested. I am curious to know in what the “professional historians of mathematics” are interested.

5.2. *Centaurus*

It took the editor only six days to reach a decision—I am grateful for that. I accept that the paper “discusses a very specialized topic.” I do not agree that “there are no references to broader history of science literature”—at least I refer to Russo (2004) and Torretti (1984). Admittedly they are few.

I probably did not observe that *Centaurus* keeps a rather popular profile, although Jesper Lützen published a pretty advanced paper there in 2010, a fact which actually made me think that the journal could take my paper. His paper is one of very few papers on mathematics during the last years, thus an exception—a misleading exception.

5.3. *Archive for history of exact sciences*

1. “the article draws on no evidence other than Euclid’s Elements to illuminate how straight lines and other geometrical objects and relations were understood in the period.” This is true, but what other scientists should I study?

Bo Göran Johansson (private communication of 2012 December 11) mentions as possible names Apollonius of Perga and Archimedes, but first of all Aristotle. Aristotle’s *Prior Analytics* seems to contain only syllogisms; *Posterior Analytics* might contain something; *Metaphysics* maybe gives more hope. Books $\mu = \text{XIII} = 13$ and $\nu = \text{XIV} = 14$ contain mathematics. Bo Göran is not sure that would give much, though. Euclid is outstanding; the argument to include others seems quite weak.

2. “it is too much of an anachronism to compare Euclid’s treatment with a modern discussion of the projective plane.” This is a terrible misunderstanding. I never compared Euclid with “a modern discussion of the projective plane.” The reason I talk about the projective plane is this. To prove that axioms A, B, and Γ do not imply a statement Σ , one finds a model M in which A, B, and Γ are true, but Σ is false. This is how Lobačevskij, Bolyai and Gauss could prove that the Postulate of Parallels cannot be deduced from the other axioms. Similarly we need a model in which the axioms are true, but not Proposition 16. One such model is the projective plane. There may exist others.

The discussion on the projective plane has no implications on the understanding of ancient mathematics. It is just there as a device to prove a statement.

I did not believe that this point could be misunderstood, but obviously it can. This incredible fact made me insert an elementary explanation of how to prove the independence of an axiom.

3. “I would add that the historical approach would be attempt to see how issues about lines in the plane were formulated, and then revised, as the concept of a projective plane was itself worked out in the course of the 19th century.” This strikes me as a bogus argument, and makes me very disappointed. To study how the concept of the projective plane was developed in the 19th century would be a completely different study—it must be possible to study Euclid without going into that. It amounts to nothing less than saying: your paper is about x ; it should be about y .

6. Sabetai Unguru

Staffan Rodhe mentioned the name of Sabetai Unguru and the fight he started in the 1970s on how to approach and understand older mathematics: Unguru insisted that one should not use modern concepts in that effort.

Sabetai Unguru published an article on “the need to rewrite the history of Greek mathematics” (1975/76). He “resents very much the traditional commentaries on Greek mathematics, especially those on Greek geometric algebra and its potential connection with Babylonian algebra” (Kirsti Moller Pedersen in her review of this article in *MathSciNet*).

Earlier Árpád Szabó (1969:458) had denied the algebraic nature in Greek geometry, although in a much more civilized way:

Aber es kann gar keine Rede davon sein, daß diese Theoreme ursprünglich »*algebraische Sätze*« oder Lösungen für »*algebraische Aufgaben*« gewesen wären. Nein, diese sind alle — sowohl die Sätze wie auch die Aufgaben — *rein geometrischen Ursprungs*. (Szabó 1969:458)

I guess an explanation to Unguru's opinion can be found in his own paper:

The situation is particularly scandalous in the history of ancient and medieval mathematics. It is in truth deplorable and sad when a student of ancient or medieval culture and ideas must familiarize himself first with the notions and operations of *modern* mathematics in order to grasp the meaning and intent of modern commentators dealing with ancient and medieval mathematical texts. (Unguru 1975/76:68)

I think this quotation is revealing. Of course, if you do not know the modern concepts, it is hard to understand descriptions using them. I tend to believe that this is the real reason behind his anger.

But for those who already happen to know some “modern” mathematics an effective way of explaining historical ideas is to use the modern ones. Of course one should never even hint that the old used the modern notions or modern terms. It is a pedagogical tool and may of course also reveal early signs of an idea made explicit much later.

There are two instances of this in my paper. One is when I speak of equivalence classes in connection with straight lines. Two points uniquely determine a straight line segment in the Euclidean plane, and a straight line segment determines two points, viz. its endpoints. In this way we have existence and uniqueness. But as soon as an *eutheia* means a straight line, and we at the same time do not allow actual infinity, we are in trouble. You extend a straight line segment, and you get a new straight line segment, but in some way or another the two segments represent the same line. This poses a problem of uniqueness. I do not know how Euclid handled this, but I am pretty sure that he must have thought of it in some way, since the problem is so obvious.

The other case is when I speak about the projective plane. I should have made sure that this is just to prove a statement, but I thought this would not be necessary.

It could be that Unguru's anger still plays some role, and that several of today's journals are under this influence and therefore shy away from using modern notions.

Bartel Leendert van der Waerden (1903–1996) and André Weil (1906–1998) replied to Unguru's article; see van der Waerden (1975/76) and Weil (1978/79). Unguru returned in (1979).

Weil's plenary talk in Helsinki in 1978, published in Weil (1980), is highly relevant and contains some weak echos of this affair, although Unguru is not mentioned. However, the following is a direct reply to Unguru.

On the other hand, when quadratic equations, solved algebraically in cuneiform texts, surfaces again in Euclid, dressed up in geometric garb without any geometric motivations at all, the mathematician will find it appropriate to describe the latter treatment as “geometric algebra” and will be inclined to assume some connection with Babylon, even in the absence of any concrete “historical” evidence. No one asks for documents to testify to the common origin of Greek, Russian and Sanskrit, or objects to their designation as indo-european languages. (Weil 1980:228–229)

7. The fourth submission: Finally accepted

After an informal contact on 2013 April 07 with Ulf Persson, the Editor-in-Chief of *Nordisk Matematisk Tidskrift (Normat)*, he encouraged me to submit the paper to that journal. I did so on 2013 September 20, and it was formally accepted on 2013 November 09. It was published on 2015 April 02.

In the submitted version I added a new section, “2. Approaches to this paper,” where I mention the standard method to prove that an implication does not hold. To prevent any accusation that equivalence classes constitute an anachronism, I added a paragraph in Subsection 4.8.

There are some other journals in the area: *Historia Mathematica* and *BSHM Bulletin. Journal of the British Society for the History of Mathematics*, which Jan P. Hogendijk recommended on 2012 July 13 (but which I did not try). Are these journals under the influence of Unguru’s anger? There is also a Brazilian journal: *Revista Brasileira de História da Matemática*. Perhaps it is sufficiently far away.

All in all, I am most grateful to Ulf Persson for accepting an article that does not fit into the little boxes that many journal editors use as criteria, the IIO editors as Euclid would have said.

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