Mikael Passare 1959–2011

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Mikael Passare lecturing at the Nordan meeting in Mariehamn, Åland, 2008 (Photo: Ragnar Sigurðsson)

Mikael Passare died from a sudden cardiac arrest in Oman in the evening of 15 September 2011. His next of kin are his wife Galina Passare, his son Max and his daughter Märta.

Mikael was born in Västerås, Sweden, on 1 January 1959 and pursued a rapid and brilliant career as a mathematician. He started his studies at Uppsala University in the Autumn of 1976 while still a high-school student, merely seventeen and a half. He finished high school in June 1978 in Västerås and gave his first seminar talk in November 1978 at Uppsala University, where he got his Bachelor's degree in 1979 and where he also worked as an assistant. He was then a PhD student with me as his advisor and he presented his thesis on 15 December 1984. He was appointed full professor at Stockholm University on 1 October 1994.

He spent four academic years in four different countries: 1980–81 at Stanford University; 1981–82 at Lomonosov University in Moscow; 1986–87 at Université Pierre et Marie Curie, Paris VI (he was also often at Orsay, Université Paris-Sud 11); and 1992–93 at Humboldt-Universität zu Berlin. He was a guest professor in France on several occasions: at Toulouse (June 1988), Grenoble (April 1992), Bordeaux (May 1992), Paris VII (March 1993), Lille (April 1999) and Bordeaux again (June 2000).

Mikael was awarded the Lundström–Åman Scholarship during the Autumn Semester of 1984 and the Spring Semester of 1985, the Marcus and Marianne Wallenberg Prize in 1988, the Lilly and Sven Thuréus Prize in 1991 and the Göran Gustafsson Prize in 2001.

Mikael was much appreciated as a researcher and teacher and was very active outside the university. He was Head of the Department of Mathematics at Stockholm University from January 2005 through August 2010 and then Director of the newly created Stockholm Mathematics Center, which is a collaboration between Stockholm University and the Royal Institute of Technology. When Burglind Juhl-Jöricke and Oleg Viro resigned from Uppsala University on 8 February 2007, he arranged for a guest professorship for Burglind at Stockholm University and was one of the organisers of a big conference in honour of Oleg, *Perspectives in Analysis, Geometry and Topology*, at Stockholm University over seven days, 19–25 May 2008.

As Chair of the Swedish National Committee for Mathematics, he led the Swedish delegation to the General Assembly of the International Mathematical Union in Bangalore, Karnataka, India, in August 2010.

Mikael Passare was Deputy Director for Institut Mittag-Leffler, Djursholm, Sweden, from 2010. He was very much appreciated for his activity there, which included organising the Felix Klein Days for teachers and a research school for high-school students.

At the time of his death, Mikael was President of the Swedish Mathematical Society and also a member of the Committee for Developing Countries (CDC) of the European Mathematical Society. His activity for mathematics in Africa is described in a later section.

Mikael's nine PhD students

Mikael served as advisor of nine PhD students who successfully completed their degrees. They are registered in the *Mathematics Genealogy Project* and are: Yang Xing, PhD 1992, Mikael Forsberg 1998, Lars Filipsson 1999, Timur Sadykov 2002, Hans Rullgård 2003, Johan Andersson 2006, Alexey Shchuplev 2007, David Jacquet 2008 and Lisa Nilsson 2009.

Mikael's mathematics

Mikael soon became known as an eminent researcher in complex analysis in several variables and his thesis was an important breakthrough with new results in residue theory. Its title was *Residues, Currents, and Their Relation to Ideals of Holomorphic Functions* and it was later published in the *Mathematica Scandinavica*.

Residue theory in several variables is a notoriously difficult part of complex analysis. Mikael's work was inspired by that of Miguel E. M. Herrera (1938–1984). Miguel and I were together at the Institute for Advanced Study in Princeton during 1965–66 and it was there that I learned about residues from him. His results, which culminated in a paper by Herrera and Lieberman and a much quoted book by Coleff and Herrera, published in 1971 and 1978, respectively, were well known long before these publications. I could somehow serve as mediator to Mikael for this interest without doing much research on residues myself.

Also, Alicia Dickenstein, who was a student of Miguel and got her PhD at Buenos Aires in 1982, knew this theory very well and soon came into contact with Mikael. As for integral formulas, Mikael took advice from Bo Berndtsson, already then a renowned expert in that field.

Mikael Passare (age 22), Jean François Colombeau, Leif Abrahamsson, and Urban Cegrell in March or April 1981 (Photo: Christer Kiselman)

Another important person for Mikael's mathematical development was Gennadi Henkin. They met in Moscow during 1981-82 and several times in the period 1984–1990 and then in Paris and Stockholm in 1991–2010.

While residues in one complex variable have been well understood for a long time, the situation is quite different in several variables. There were pioneers like Henri Poincaré (1854–1912) and Jean Leray (1906–1998), and Alexandre Grothendieck developed a residue theory in higher dimensions but it was quite abstract. Through the work of Miguel Herrera, François Norguet and Pierre Dolbeault the theory could be linked to distribution theory, which had been developed by Laurent Schwartz (1915–2002), and that was the road that Mikael continued to follow. He worked intensively with August Tsikh, both on residue theory and amoebas.

Residues in several variables

Let f and g be holomorphic functions of n complex variables. The *principal value* $\mathrm{PV}(f/g)$ of f/g is a distribution defined by the formula

$$\left\langle \mathrm{PV}\Big(\frac{f}{g}\Big), \varphi \right\rangle = \lim_{\epsilon\to 0} \int_{|g|>\epsilon} \frac{f\varphi}{g} = \lim_{\epsilon\to 0} \int \frac{\chi f \varphi}{g}, \qquad \varphi \in \mathcal{D}(\mathbf{C}^n),$$

where $\chi = \chi(|g|/\epsilon)$ and χ is a smooth function on the real axis satisfying $0 \le \chi \le 1$ and $\chi(t) = 0$ for $t \le 1$, $\chi(t) = 1$ for $t \ge 2$.

The *residue current* is $\bar\partial\,\mathrm{PV}(f/g)$. Can the products

$$(\mathrm{PV}(f_1/g_1))(\mathrm{PV}(f_2/g_2)), \quad \Big(\bar\partial(\mathrm{PV}(f_1/g_1))\Big)(\mathrm{PV}(f_2/g_2))$$

and other similar products be defined?

Mikael's construction of residue currents goes as follows. Take $f = (f_1, \dots, f_{p+q})$, $g = (g_1, \dots, g_{p+q})$, two $(p+q)$-tuples of holomorphic functions, and consider the limit

$$\lim_{\epsilon_j\to 0} \frac{f_1}{g_1} \cdots \frac{f_{p+q}}{g_{p+q}} \bar\partial\chi_1 \wedge \cdots \wedge \bar\partial\chi_p \cdot \chi_{p+1} \cdots \chi_{p+q},$$

where $\chi_j = \chi(|g_j|/\epsilon_j)$ and the ϵ_j tend to zero in some way.

Coleff and Herrera took $q = 0$ or 1 and assumed that ϵ_j tends to zero much faster than ϵ_{j+1}, which in this context means that $\epsilon_j/\epsilon_{j+1}^m \to 0$ for all $m \in \mathbf{N}$ and $j = 1, \dots, p+q-1$; thus it is almost an iterated limit. This gives rise to the strange situation that, in general, the limit depends on the order of the functions (and is not just an alternating product).

Mikael took instead $\epsilon_j = \epsilon^{s_j}$ for fixed $s_1, \dots, s_{p+q}$. The limit, which will be written as $R^pP^q[f/g](s)$, where we now write $[\dots]$ for the principal value, does not exist for arbitrary s_j. But he proved that if we remove finitely many hyperplanes then $R^pP^q[f/g](s)$ is locally constant in a finite subdivision of the simplex

$$\Sigma = \Big\{ s \in \mathbf{R}^{p+q};\ s_j > 0, \quad \sum s_j = 1 \Big\},$$

so that the mean value

$$R^pP^q\Big[\frac{f}{g}\Big] = \fint_\Sigma R^pP^q\Big[\frac{f}{g}\Big](s)$$

$$= \bar\partial\left[\frac{f_1}{g_1}\right] \wedge \cdots \wedge \bar\partial\left[\frac{f_p}{g_p}\right] \cdot \left[\frac{f_{p+1}}{g_{p+1}}\right] \cdots \left[\frac{f_{p+q}}{g_{p+q}}\right]$$

exists. This is the product of p residue currents and q principal-value distributions.

For complete intersections, i.e., when the set of common zeros of $f_1, f_2, \dots, f_p$ has maximal codimension, Mikael established a division formula with remainder term:

$$h = \sum_1^p g_jf_j + h \cdot \mathrm{Res},$$

where Res is the residue current, which is a factor in the remainder term $h \cdot \mathrm{Res}$ and has the property that $f_j \cdot \mathrm{Res} = 0$ for all j. This implies that h belongs to the ideal generated by $f_1, \dots, f_p$ if and only if $h \cdot \mathrm{Res}$ vanishes. This is a beautiful characterization of the ideals of holomorphic functions and explains the choice of title in several of his papers. The characterization of the ideals with the help of residues was proved independently and at about the same time by Alicia Dickenstein and Carmen Sessa.

This characterization of ideals enabled Mikael and Bo to formulate an elegant and explicit variant of Leon Ehrenpreis' Fundamental Principle; it was published in a joint paper with Bo in 1989. Later, in 2007, Mats Andersson and Elizabeth Wulcan could define a residue without the assumption of a complete intersection. In this work, a paper by Mikael, August and Alain Yger played an important role.

Mikael showed that his original definition of residues and the definition which uses meromorphic extension agree.

Lineal convexity

André Martineau (1930–1972) gave a couple of seminars on lineal convexity (*convexité linéelle*) in Nice during the academic year 1967–68 when I was there. This is a kind of complex convexity which is stronger than pseudoconvexity and weaker than convexity. Since I was of the opinion that the results for this convexity property were too scattered in the literature and did not always have optimal proofs, I suggested that Mikael write a survey article on the topic.

On the one hand, this piece of advice was certainly very good, for he found a lot of results in cooperation with his friends Mats Andersson and Ragnar Sigurðsson (Mikael's mathematical uncle). On the other hand, it was perhaps not such a good suggestion, for the survey just kept growing; two preprints started circulating in 1991 and by then they had been busy writing for a long time already. The article became a book and it did not appear until 2004. Anyway, it is thanks to André Martineau that lineal convexity came to be studied in the Nordic countries – and the book has become a standard reference.

Amoebas and tropical geometry

Mikael's later work is concerned with amoebas and coamoebas – the first publications in this field were Mikael Forsberg's thesis of 1998 and a joint paper published in 2000. The spine of an amoeba – in mathematical zoology, amoebas are vertebrates – is a tropical hypersurface. Tropical mathematics is a rather new branch of mathematics where addition and multiplication are replaced by the maximum operation and addition, somewhat similar to taking the logarithm of a sum and a product. His interest in tropical mathematics was a break with his earlier work on complex analysis, which he once compared with my switching to digital geometry.

An amoeba is a set in $\mathbf{R}^n$ defined as follows. We define a mapping

$$\mathrm{Log}\colon (\mathbf{C} \smallsetminus \{0\})^n \to \mathbf{R}^n \text{ by}$$

$$\mathrm{Log}(z) = (\log|z_1|, \log|z_2|, \ldots, \log|z_n|).$$

If f is a function defined in $(\mathbf{C} \smallsetminus \{0\})^n$ then its *amoeba* is the image under Log of its set of zeros. The term was introduced by I. M. Gelfand, M. M. Kapranov and A. V. Zelevinsky in 1994.

One can of course study the image in $\mathbf{R}^n$ of any set but zero sets of certain functions have interesting properties. An amoeba is typically a closed semianalytic subset of $\mathbf{R}^n$ with tentacles which go out to infinity and separate the components of the complement of the amoeba. The number of such components is at most equal to the number of integer points in the Newton polytope for f if f is a Laurent polynomial, in certain cases equal to the latter number.

An easy example, which Mikael himself used in his lectures, is the zero set of the polynomial $P(z, w) = 1 + z + w$ of degree one. A zero $(z, w) \in \mathbf{C}^2$ must satisfy $1 \le |z| + |w|$, $|z| \le |w| + 1$ and $|w| \le 1 + |z|$. It is easy to see that any point $(p, q) \in \mathbf{R}^2$ which satisfies the inequalities $1 \le p+q$, $p \le q+1$ and $q \le 1 + p$ is equal to $(|z|, |w|)$ for some zero (z, w) of P. (A useful observation here is the fact that the corresponding strict inequalities are the exact conditions under which there exists a triangle with side lengths 1, p and q.) The amoeba of P is then given by the three inequalities $1 \le e^x + e^y$, $e^x \le e^y + 1$ and $e^y \le 1 + e^x$.

A *coamoeba* is defined analogously but with the mapping Log replaced by the mapping $\mathrm{Arg}(z) = (\arg z_1, \arg z_2, \ldots, \arg z_n)$. Mikael wanted to establish formally the duality between amoebas and coamoebas and he started to write a paper with Mounir Nisse, which Mounir is now finishing.

In a little paper published in the *Monthly* in 2008, which is indeed a gem, Mikael shows how the concept of an amoeba can be used to show the well known formula $\zeta(2) = \sum_1^\infty 1/n^2 = \pi^2/6 \approx 1.644934$ (the so-called Basel problem).

The Pluricomplex Seminar

I started a seminar series in Uppsala in the 1970s, later to become known as *The Pluricomplex Seminar* – a name I borrowed from Jean-Pierre Ramis. Mikael gave his first lecture in the seminar during the Autumn Semester of 1978. He reported on chosen sections of the little book by Lev Isaakovič Ronkin (1931–1998), *The Elements of the Theory of Analytic Functions of Several Variables*, which had been published in Russian (in 2,700 copies) in Kiev the year before and cost 93 kopecks. The task was a part of the examination for the course *Mathematics D*. He gave a total of 29 seminar talks over the period 1978–2010.

Håkan Samuelsson, Elin Götmark, Elizabeth Wulcan, Mikael Passare and Liz Vivas at Institut Mittag-Leffler, Spring 2008 (Photo: Ragnar Sigurðsson)

Originally, the seminars took place at Uppsala with a lecture almost every week. From the Spring Semester of 1999 onwards, when Mikael had become well established as a professor at Stockholm, they became a joint activity for Uppsala University, Stockholm University and the Royal Institute of Technology (KTH). From 2007, when I had switched to digital geometry, mathematical morphology and discrete optimization, and Burglind Juhl-Jöricke had left Uppsala University, it became an activity exclusively in Stockholm.

The *Nordan* Meetings

Together with Mats Andersson and Peter Ebenfelt, Mikael Passare initiated a series of encounters on complex analysis in the five Nordic countries. Mikael and Peter organised the first conference, which took place in Trosa, Sweden, 14–16 March 1997, and Mats organised the second, in Marstrand, Sweden, 24–26 April 1998. Following a voting procedure at the end of the first meeting, these annual meetings were named *Nordan*[1] – a clear reference to *Les Journées complexes du Sud*, which over a long period have taken place in the south of France.

Nordic meetings like these were something that Mikael and Mats had discussed and planned for many years. And the initiative turned out to be a long lasting success: the 15th encounter took place in Röstånga in southern Sweden, 6–8 May 2011; the 16th in Kiruna in northern Sweden, 11–13 May 2012.

Africa

Mikael Passare was a Member of the Board of the International Science Programme (ISP), Uppsala, and a Member of the Board of the Pan-African Centre for Mathematics (PACM) in Dar es-Salaam, Tanzania. He was a driving force in the creation of this Pan-African Centre, which is a collaborative project between Stockholm University and the University of Dar es-Salaam.

Mohamed E. A. El Tom, Chairman of the Board of PACM and a member of the Reference Group for Mathematics of ISP, says he is confident that had it not been for Mikael PACM would have remained a mere idea in the head of its initiator, i.e., in Mohamed's head.

Mikael's last assignment was to chair and constitute a search committee for the Director of the Centre. He accepted the charge and promised to respond with detailed ideas upon his return from his trip to Dubai, Oman and Iran.

Mikael's commitment and enthusiasm for the Centre was unsurpassed. He was confident that the grand objective of establishing a world-class Centre of Mathematics in Africa is attainable.

Sonja Kovalevsky

The chair which Mikael Passare held was the one which was created for Sonja Kovalevsky (03/15 January 1850 – 10 February 1891). An earlier incumbent for seven years (1957–1964) was Lars Hörmander, Mikael's mathematical grandfather. Mikael was proud of having been given Sonja's chair. He is buried not far from her grave.

Exactly 150 years after Sonja's birth, on 15 January 2000, Mikael organised a symposium to her memory. It was held in the *Aula Magna* of Stockholm University. Among the invited speakers were Agneta Pleijel, Roger Cooke and Ragni Piene.

Languages and music

Mikael knew many languages. His Russian was "really perfect!" according to Timur Sadykov. "He spoke Russian perfectly, so it was totally impossible to recognise his Swedish origin," said Andrei Khrennikov. He took a course in French corresponding to 30 ECTS credits at Stockholm University before going to Paris in 1986–87. He learned some Fijian when he visited the Republic of Fiji.

His knowledge of German was very good, although he had not studied that language in high school. He also studied Finnish and spoke the language so well that he was interviewed on the Finnish-language *Sisuradio* in Sweden.

Spanish and Italian he knew enough to get along. He was recently in Italy and Spain with Anders Wändahl and never talked English when visiting a restaurant or when asking for directions in the street. He could also speak some Polish and Bulgarian.

Finally, he studied Arabic and could at least read that language. Maybe Arabic would have been his next project.

Mikael loved classical music; in his teens he sold his bicycle in order to buy a piano. He played clarinet and flute. He composed a piece for clarinet, which was played in a theatre in Stockholm. His last love was an instrument called theremin.[2] He dreamed about being able to play it.[3]

A "Swedish Classic"

Mikael performed what is known as a "Swedish Classic" in 1989. It consists of four parts, which have to be completed within a 12-month period: (1) One of the ski runs, the Engelbrekt Run (60 km) and the Vasa Run / Open Track (90 km); (2) Going around Lake Vättern on bicycle (300 km); (3) The Vansbro Swim (3 km); (4) The Lidingö Run, running (30 km). Mats Andersson remembers that he claimed the cycling to be the most painful of the four, noting the chafing after so many hours on the saddle.

A passionate traveller

Mikael was a passionate traveller. He visited 152 countries. When he and I, together with several other Swedish mathematicians, were invited in September 2006 to celebrate the 20th anniversary of the *Groupe Inter-Africain de Recherche en Analyse, Géométrie et Applications* (GIRAGA) and after that to participate in the *First African-Swedish Conference on Mathematics*, both in Yaoundé, Cameroon, he first visited the Central African Republic and continued afterwards to Equatorial Guinea and Gabon; thus he got four new countries on his list – assuming that he had not been to any of these before – while I got only one.

The United Arab Emirates and Oman turned out to be the last ones. Land number 153 should have been Iran: he planned to arrive at Tehran Imam Khomeini International Airport at 21:25 on 17 September, as he wrote on 15 September 2011, the last day of his life, to mathematicians in Tehran. Siamak Yassemi, Head of the School of Mathematics, University of Tehran, was ready to meet him there.

Finally

Mikael's significance goes much beyond his own research. Many people have testified to his positive view of life, his humour and to his genuine interest in people he met. He was an unusually stimulating partner in discussions: listening, inspiring and supportive, in professional situations as well as private ones.

For Mikael's friends and colleagues around the world his unexpected departure is a severe loss.

For an unabridged obituary and a manuscript entitled "Questions inspired by Mikael Passare's mathematics" see the webpage www.math.uu.se/~kiselman/passareinmemoriam.html.

Notes

1. This is the name in Swedish of a chilly wind from the north but also reminds us of the original purpose: to promote Nordic Analysis.
2. Терменвокс, which was invented by Лев Сергеевич Термен, Léon Theremin (1896–1993).
3. At his funeral on 28 October 2011, *Dance in the Moon* was played on CD; the performer was Lydia Kavina, a leading thereminist.