Homework 2. Due Dec. 1st

Problem 1.

The Bessel function J_ν is one of the solutions to the Bessel equation

$$x^2 y'' + xy' + (x^2 - 1)y = 0$$

Use the method of Frobenius to obtain the second linearly independent solution.

Problem 2. Find a Green's function for the system

$$y'' = f, \quad y(0) = y(1) = 0.$$

Check your answer by verifying that it gives $x(x - 1)$ s the solution when $f = 2$.
Evaluate the eigenvalues and eigenfunctions of

$$y'' = \lambda y, \quad y(0) = y(1) = 0,$$

and consequently find an orthonormal basis of $L^2[0, 1]$.

Problem 8.1 (i) through (iv) from HSD (page 184) For each of the following systems
a) Find all of the equilibrium points and describe the behaviour of the assiciated liniear system.

b) Describe the phase portrait of the nonliner system.

c) Does the linearized system accurately describe the local behavior near the equilibrium points?

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\begin{aligned}
(i) &\quad x' = \sin x, \quad y' = \cos y, \\
(ii) &\quad x' = x(x^2 + y^2), \quad y' = y(x^2 + y^2), \\
(iii) &\quad x' = x + y^2, \quad y' = 2y, \\
(iv) &\quad x' = y^2, \quad y' = y
\end{aligned}
$$