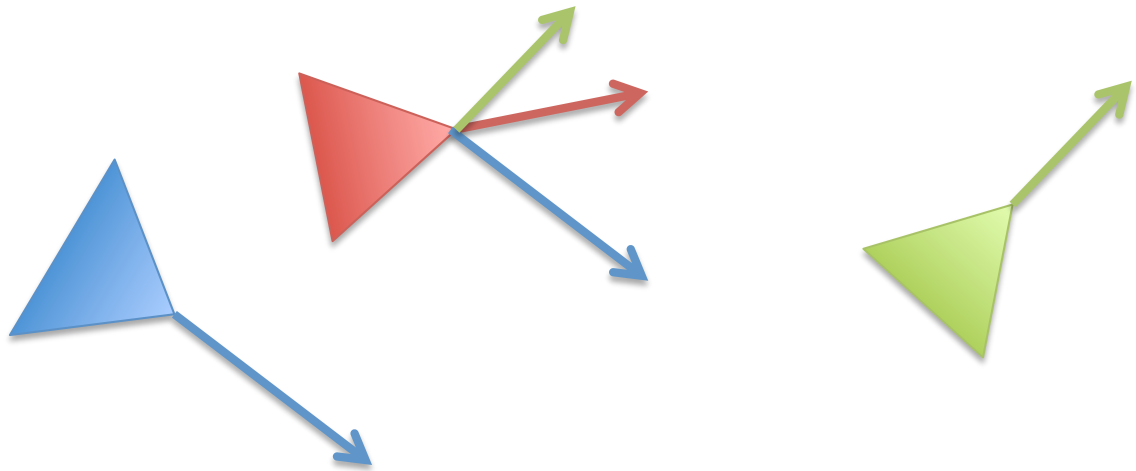
Self-Propelled Particles




SPPs

- ?: What are the underlying mechanisms ruling the particles moving?
- ?: How can we measure and compare different systems and their states?
- ?: Can we predict the collective motion features?

General SPP Model



$$\vec{x_i}(t+1) = \vec{x_i}(t) + \vec{u_i}(t+1)$$

$$\vec{u_i}(t+1) = f(\vec{u_i}(t), (\vec{x_j}(t), \vec{u_j}(t)), e)$$

where j is particle i's neighbor, e is some random effect

Attraction

- Individuals change their velocity according to the positions of their neighbors.
- An example in one dimension:

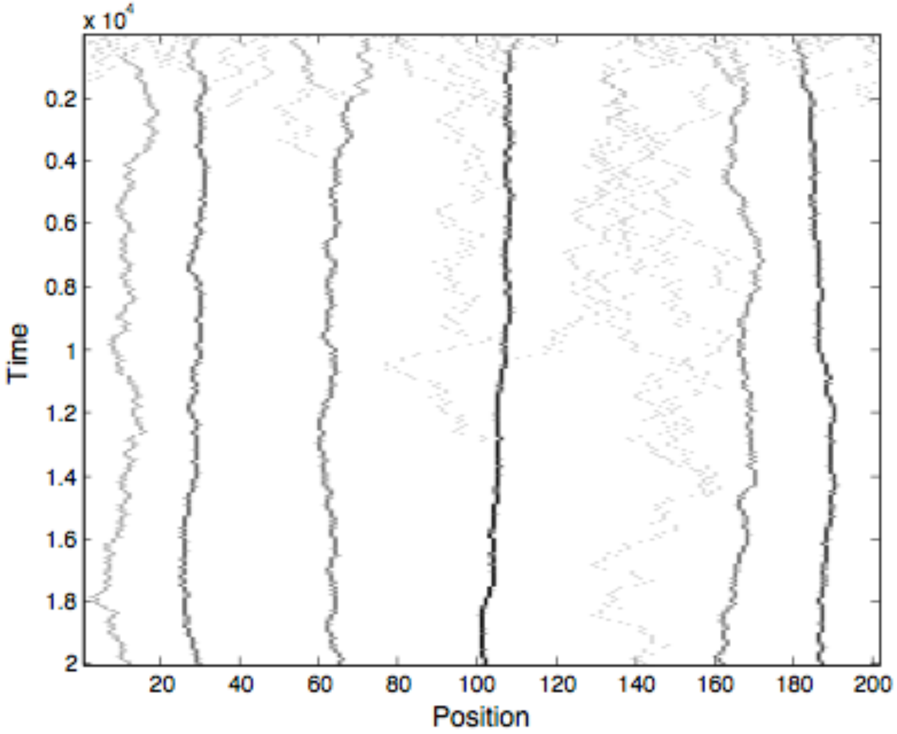
$$x_i(t+1) = x_i(t) + v_0 u_i(t+1)$$

$$u_i(t+1) = \alpha u_i(t) + (1-\alpha) s_i(t) + e$$

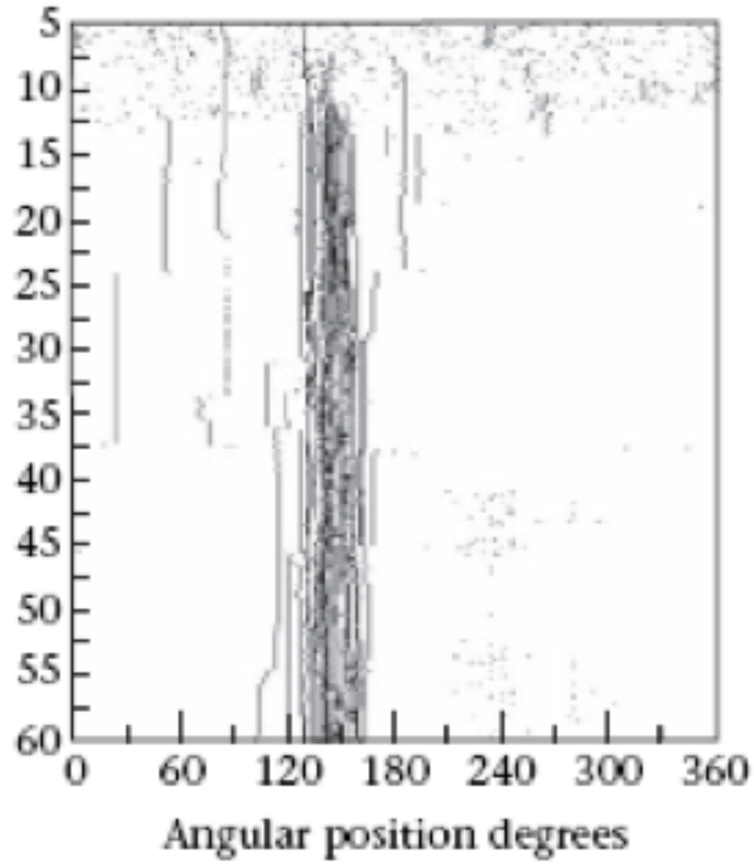
$$s_i(t) = \frac{1}{N_i(t)} \sum_j sign\{x_j(t) - x_i(t)\}$$

e is a random number uniformly distributed in [-η/2, η/2], $N_i(t)$ is the number of neighbors individual i has at time t.

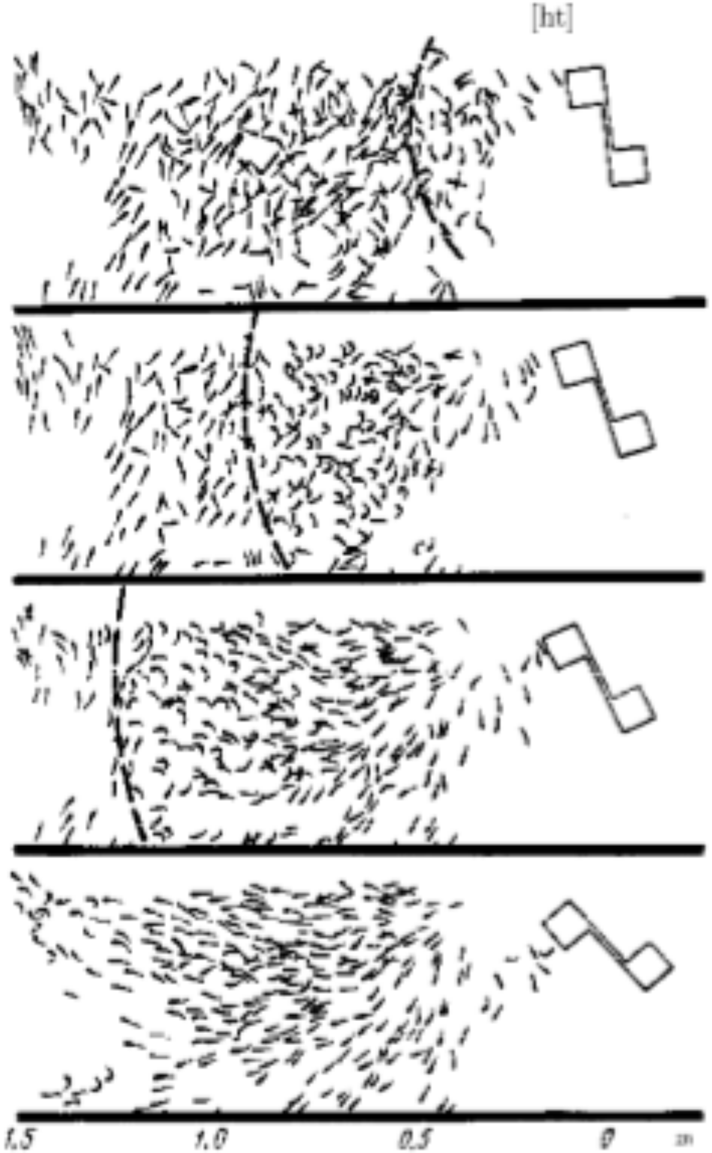
Simulation vs. Experiment



Cockroaches



Radakov’s Fish Experiment



Alignment (Vicsek's model)

- Individuals change their velocity according to the velocity of their neighbors.
- An example in one dimension:

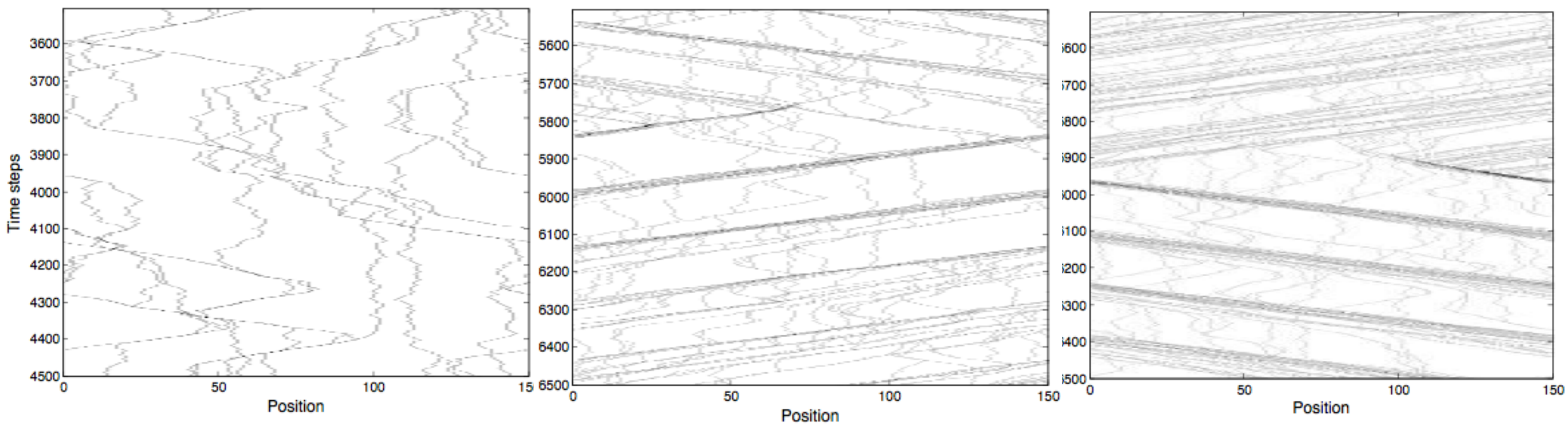
$$x_i(t+1) = x_i(t) + v_0 u_i(t+1)$$

$$u_i(t+1) = \alpha u_i(t) + (1-\alpha) s_i(t) + e$$

$$s_i(t) = G(\frac{1}{N_i}\sum_j u_j(t)) \text{ where } G(u) = \begin{cases} (u+1)/2 & \text{for u} > 0 \\ (u-1)/2 & \text{for u} < 0 \end{cases}$$

e is a random number uniformly distributed in [-η/2, η/2], $N_i(t)$ is the number of neighbors individual i has at time t.

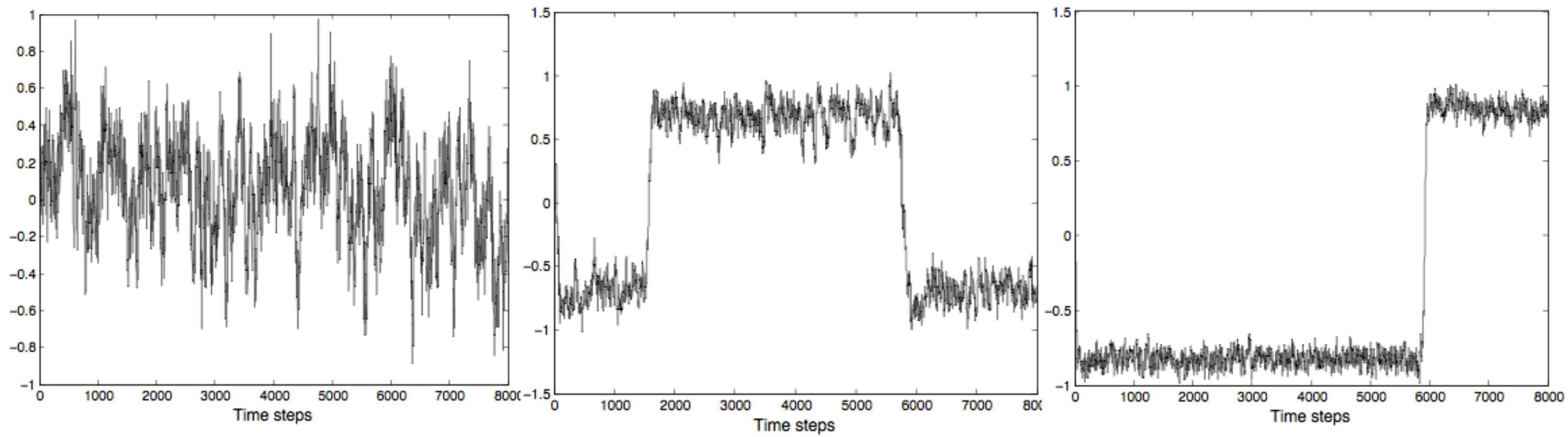
Simulation results



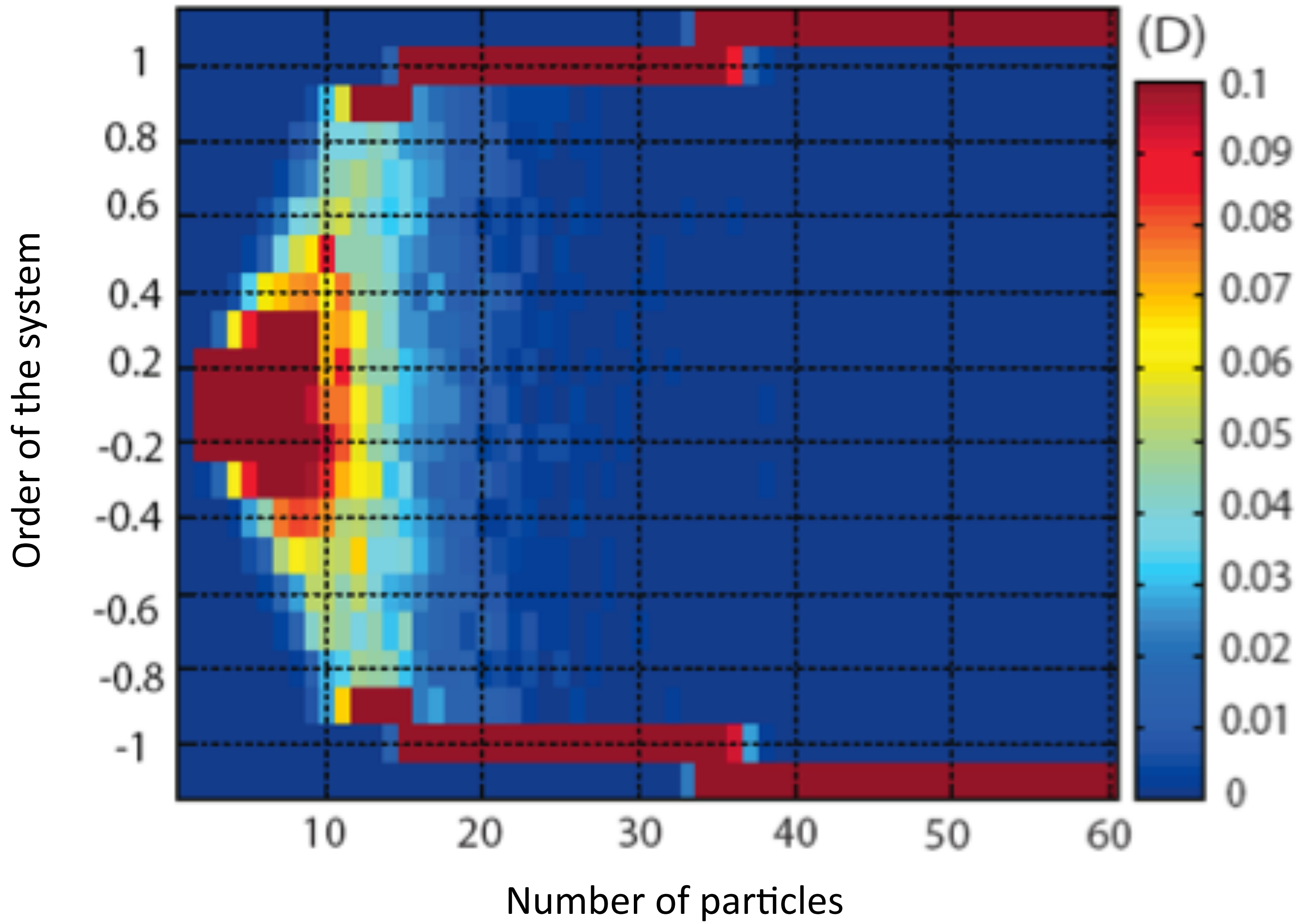
Degree of alignment

- Measure the order of the system: the average direction

$$\varphi = \frac{1}{N v_0} \sum_{i=1}^{N} \vec{v_i}$$



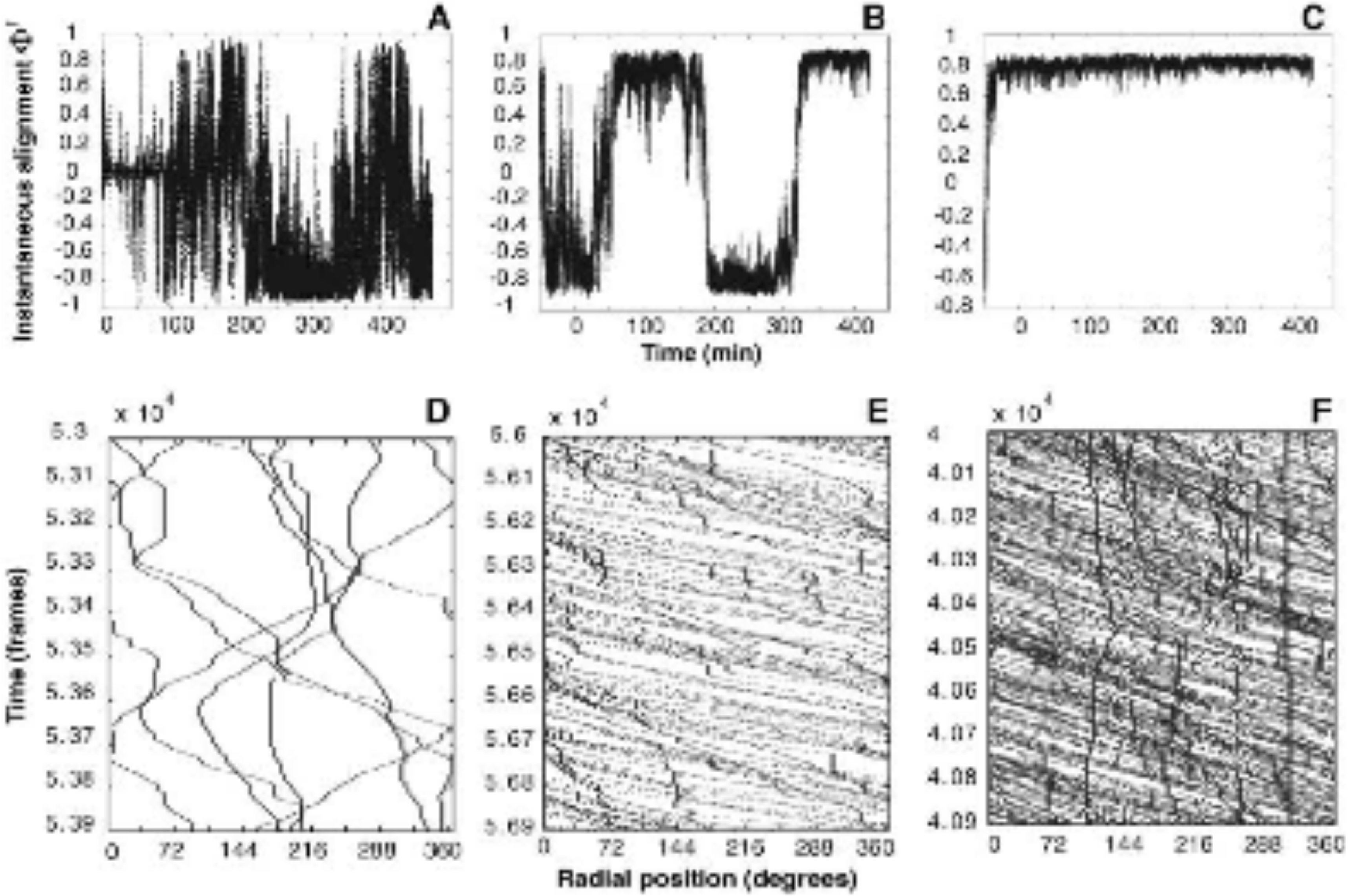
The phase transition diagram



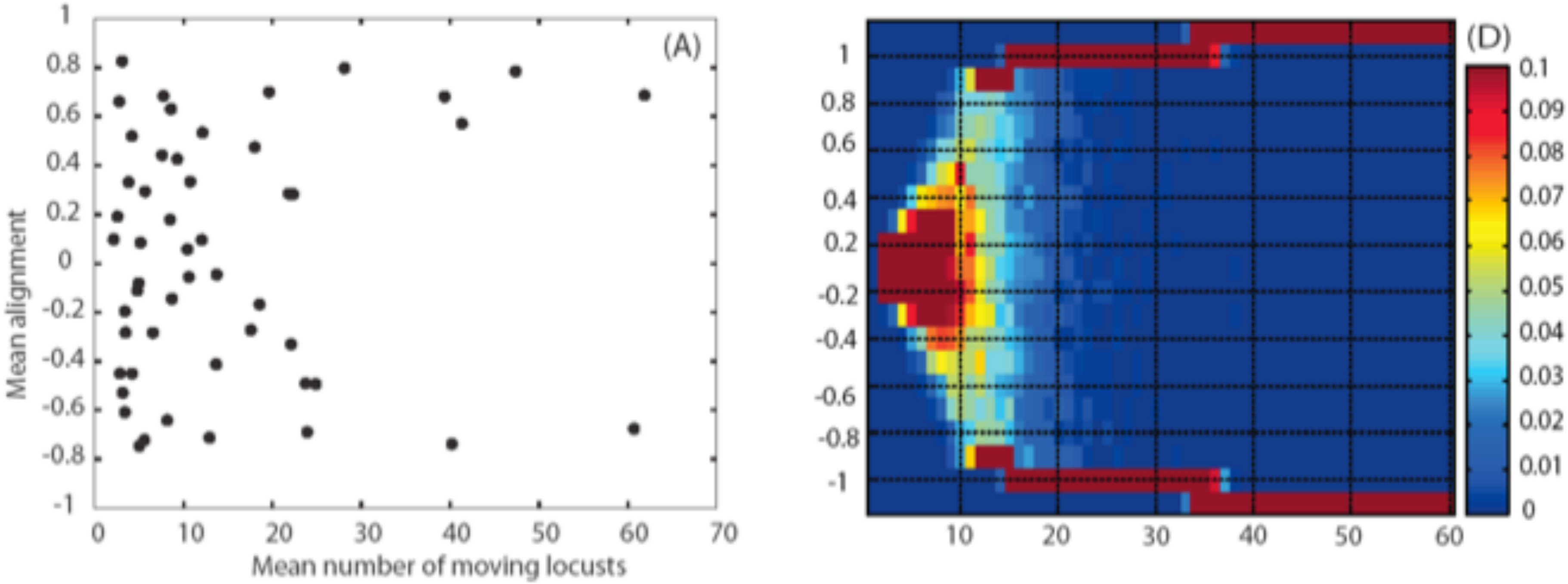
Experiments of Locusts (Buhl et al.)

- The experiment video here

Experiments Results



The phase transition diagram



Vicsek's extension of the Alignment model to 2D

- Individual i changes its direction based on the average direction of all individuals in radius R_i (including itself)

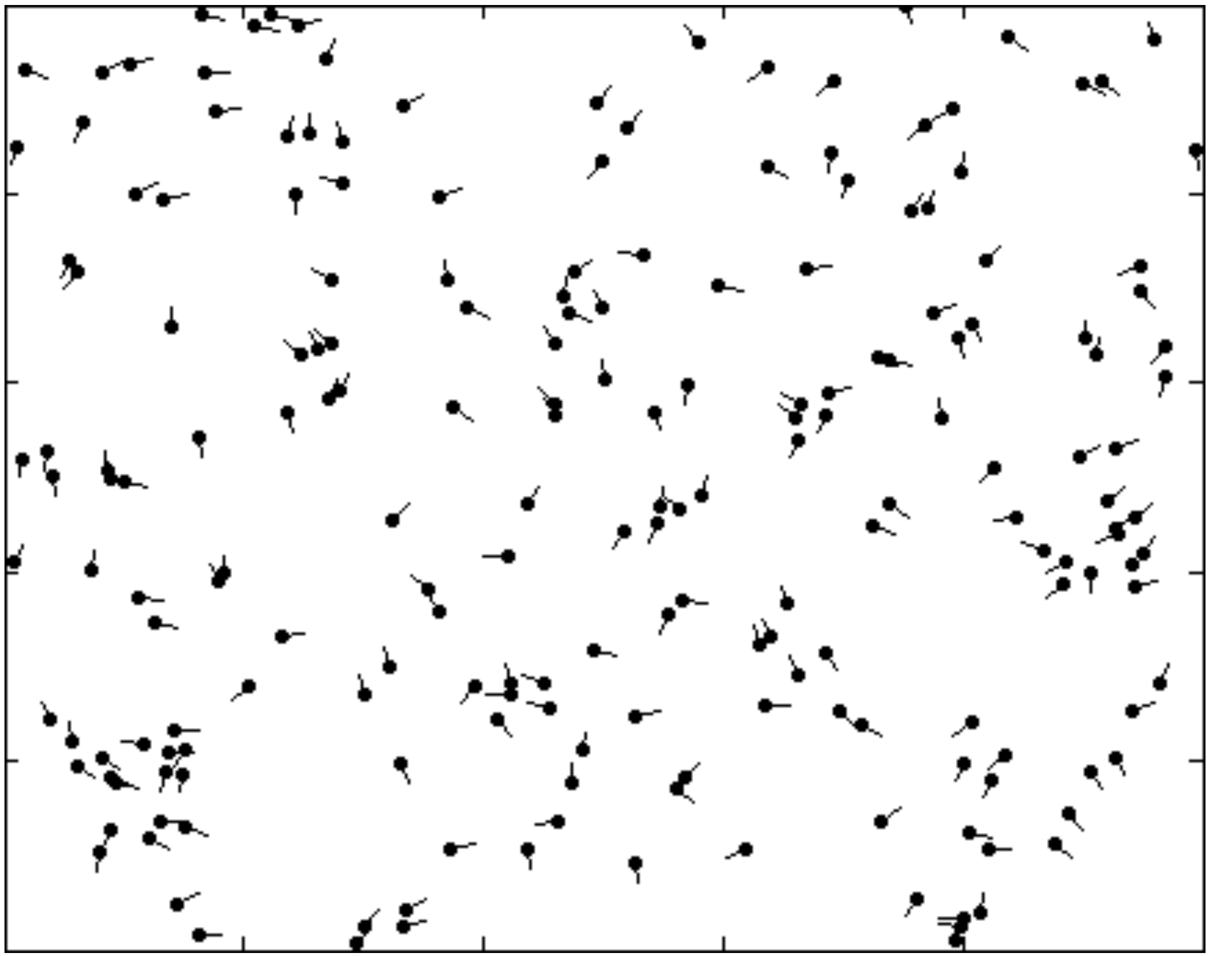
$$\vec{x_i}(t+1) = \vec{x_i}(t) + \vec{v_i}(t+1)$$

Here the velocity $\mathbf{v_i}$ is constructed to have an absolute value v_0 and direction determined by

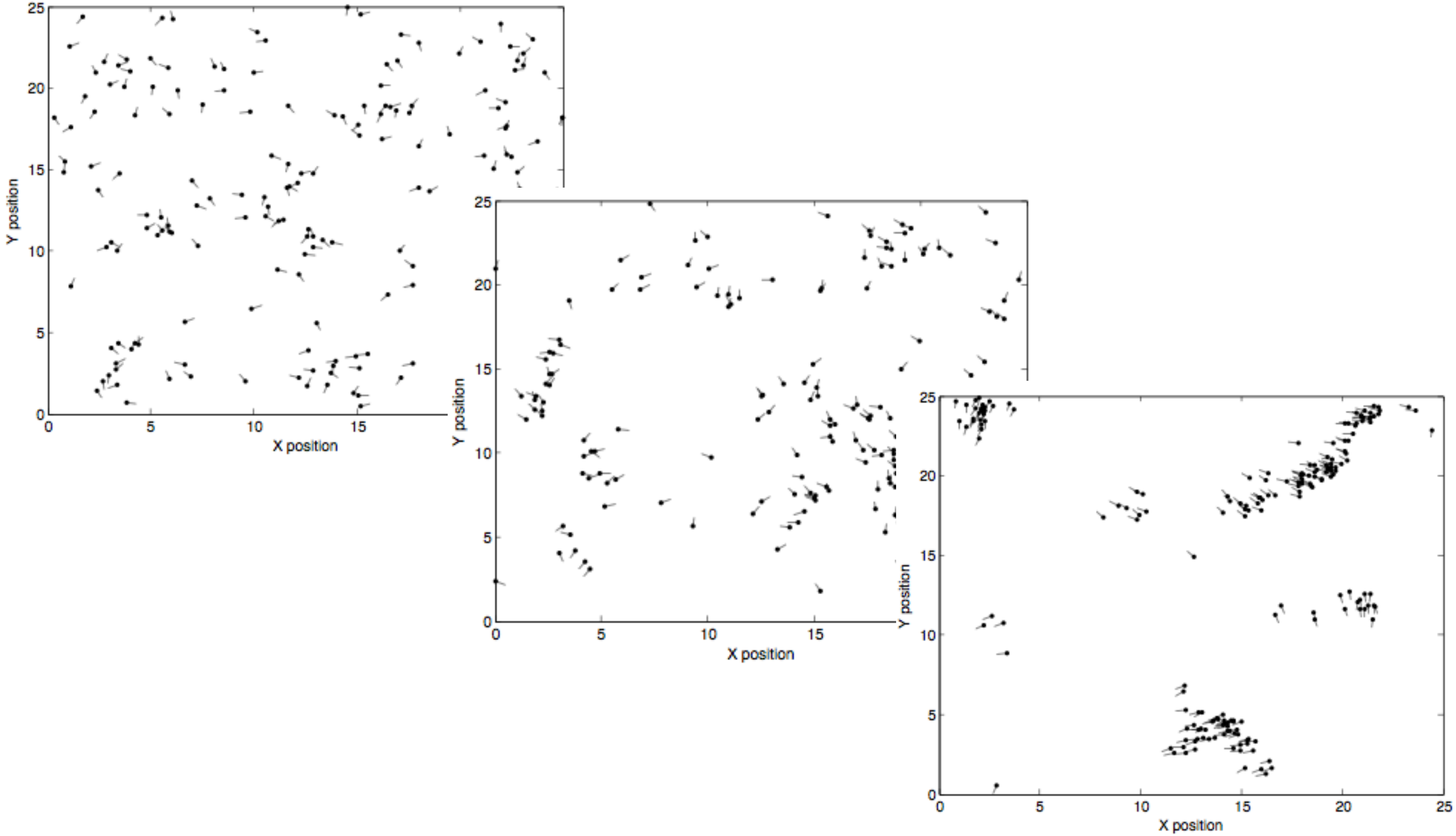
$$\theta(t+1) = \langle \theta(t) \rangle_r + \Delta\theta$$

$$\langle \theta_i(t) \rangle_r = arctan\left(\frac{\sum_j \sin\theta_j(t)}{\sum_j \cos\theta_j(t)} \right)$$

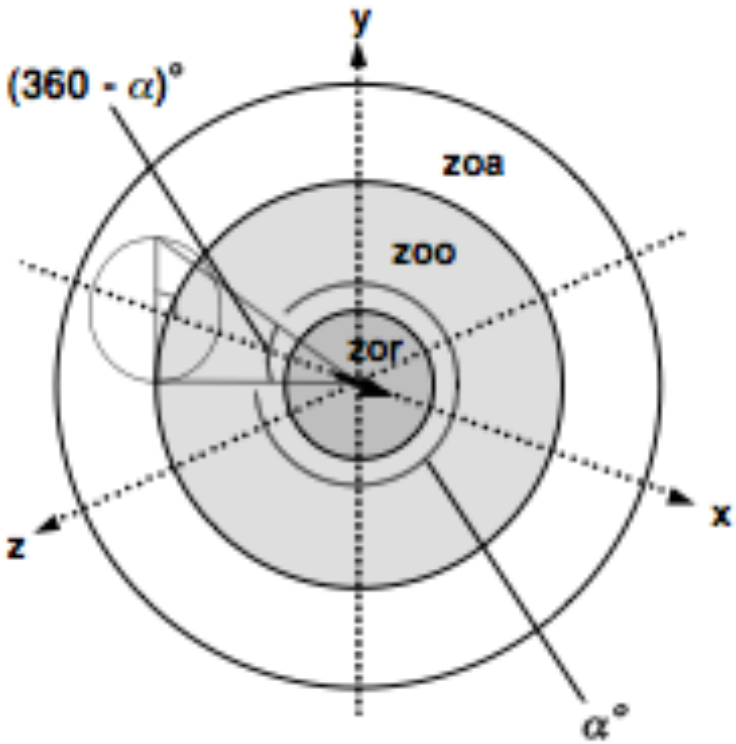
The simulation



For different noise strength



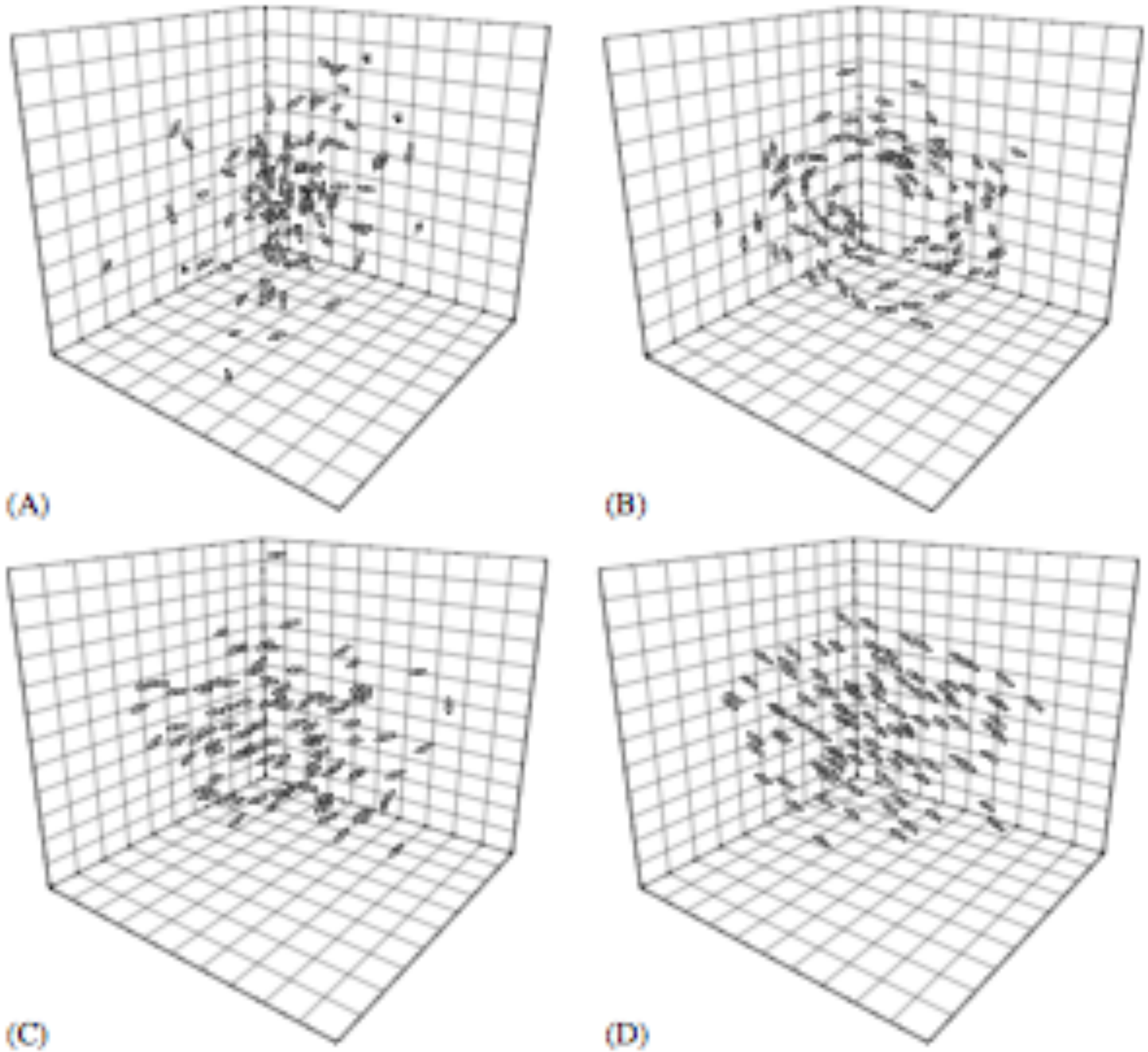
3 zones model (Couzin et al.)



$$\vec{d_r}(t+1) = -\sum_{j\neq i}^{n_r} \frac{\vec{x_j}(t) - \vec{x_i}(t)}{|\vec{x_j}(t) - \vec{x_i}(t)|}$$

$$\vec{d_o}(t+1) = \sum_{j\neq i}^{n_a} \frac{\vec{u_j}(t)}{|\vec{u_j}(t)|}$$

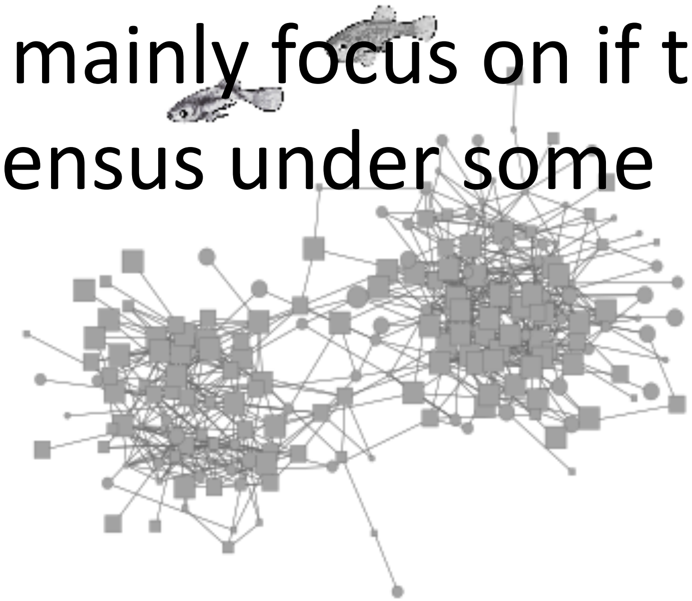
$$\vec{d_a}(t+1) = \sum_{j\neq i}^{n_a} \frac{\vec{x_j}(t) - \vec{x_i}(t)}{|\vec{x_j}(t) - \vec{x_i}(t)|}$$



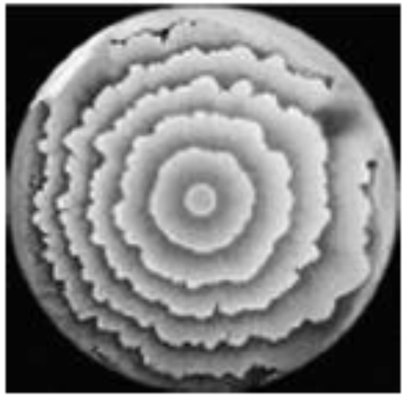
Radius of orientation zone Δr_o increasing from A to D

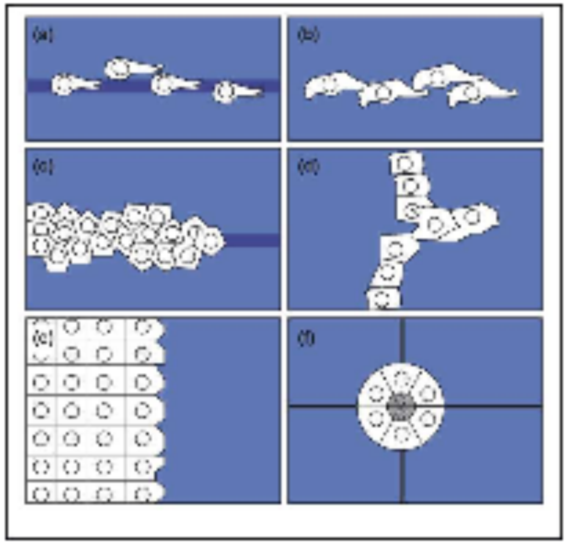
Network protocol

- The system can be represented by a graph where the nodes are particles, two nodes are connected with a link if they interact with each other (i.e they are neighbors).
- More rigorous, research mainly focus on if the system can achieve consensus under some network structures.



Other systems studied by SPP models





- Vicsek's review paper: http://arxiv.org/abs/1010.5017

Info

- Find more about SPP models in David's book: http://www.collective-behavior.com/Site/Moving_together.html
- Demos, programs, visualizations can also be found in Craig Reynolds' boids site: http://www.red3d.com/cwr/boids/.
- If you are interested in robot, check this: swarmrobot.org