Uppsala University
Department of Mathematics
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Algebraic number theory
Spring term 2012
March 15

Home assignments

First set

1. Prove that every non-trivial integral solution (x, y, z) of the Diophantine equation

$$X^2 + Y^2 = Z^2$$

satisfies $\gcd(x, y) = \gcd(x, z) = \gcd(y, z)$.

2. Determine all trivial integral solutions of the Diophantine equation

$$X^n + Y^n = Z^n,$$

where $n \in \mathbb{N} \setminus \{0\}$.

3. If a and b are elements in a commutative ring R, then we write $a|b$ (read "a divides b") to express that $ax = b$ for some $x \in R$.

Let p be a prime number, $\zeta = e^{\frac{2\pi}{p} i}$ and $a, b \in \mathbb{Z}$. Show that $a|b$ holds in $\mathbb{Z}$ if and only if $a|b$ holds in $\mathbb{Z}[\zeta]$.

4. A complex number z is called an *algebraic integer* if $f(z) = 0$ for some integral polynomial $f(X)$ with leading coefficient 1. Prove that the set $\mathcal{Z}$ of all algebraic integers satisfies $\mathcal{Z} \cap \mathbb{Q} = \mathbb{Z}$. (Later we will see that $\mathcal{Z}$ actually is a subring of $\mathbb{C}$.)

Rules. *1. Every exercise gives at most 5 points. Your assignments should be be handed in to me or my mailbox not later than thursday, 22 March, 10 a.m.*

2. Delayed exercises will in general be ignored. Exceptions are possible, but they require your explanation and my approval in advance.

3. The home assignments are compulsory in the sense that a total score of at least 50% is a necessary requirement for passing the course. A total score of at least 75% gives you 2 credit points for the written examination.