Uppsala University
Department of Mathematics
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Algebraic number theory
Spring term 2012
May 14

Home assignments

Seventh and last set

25. Is $\mathbb{Q}$ a fractional $\mathbb{Z}$-ideal? Motivate your answer!

26. Let R be a Dedekind domain, with field of fractions K. Let A be a fractional R-ideal, and set $A^{-} = \{x \in K \mid xA \subset R\}$. Show that if A is invertible, then $A^{-1} = A^{-}$.

27. Let R be a noetherian commutative ring. Let $I_0 \subset I_1 \subset I_2 \subset \ldots$ be an infinite ascending chain of ideals in R.

(a) Show that $I = \bigcup_{i \in \mathbb{N}} I_i$ is an ideal in R.

(b) Conclude that there exists an index $i_0 \in \mathbb{N}$ such that $I_{i_0} = I_i$ holds for all $i_0 \leq i$.

28. Prove that every non-empty family $\mathcal{S}$ of ideals in a noetherian commutative ring R has a maximal member $M \in \mathcal{S}$.

Every exercise gives at most 5 points. Your assignments should be handed in to me or my mailbox not later than Monday, May 21, 10 a.m. Delayed exercises will in general be ignored. Exceptions are possible, but they require your explanation and my approval in advance.