Aspects of Wireless Network Modeling based on Poisson Point Processes

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Purpose

Stochastic models for simplified wireless network

- Spatially distributed stations with emitters/receivers for transmission over a common communication channel.
- Approach based on Poisson point processes for spatial locations, signal strength, fading effects, session length
- Signal transmissions syncronized and slotted in time; one symbol per slot.
- Signal power affected by Rayleigh fading, attenuation prop. to traveled distance, and lognormal fading.
- Interference field: superposition effect of all stations.
- Signal to noise and interference ratio; compute or estimate success probability.
- Balance between node density and node interference.

Model extentions

- Modeling scenario for Rayleigh fading based on Lévy gamma subordinator processes relation to complex Gaussian waveforms (continuous time).
- Sessions which are Poisson in both space and time.
- Short-tailed or heavy-tailed random session duration times.
- Scaling approximation to analyze the fluctuations in the interference field. Brownian motion, fractional Brownian motion, etc.

Outline

Preliminary notes are available

One-slot models

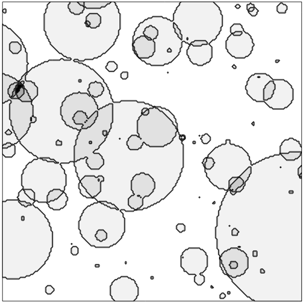
- Connectivity model
- Pathloss model
- Rayleigh fading
- Poisson calculus
- Multicast model
- Interference model
- Node density versus interference

Traffic session modeling

- Rayleigh fading over fixed session length
- Lognormal fading
- Temporal-spatial traffic sessions
- Fluctuations of interference field
- Scaling analysis

Connectivity model

Move around (receiver) in space: count number of nodes within transmission range



Center balls $B(x, r)$ of random radius r at locations $x \in \mathbf{R}^d$ of a Poisson point measure, Lebesgue intensity λdx. Each ball has independent radius R from distribution $F_R(r) = P(R \le r)$. Let $N_\lambda(dx, dr)$ be a Poisson point measure with intensity $\lambda dx\, F_R(dr)$. The number of "successful transmissions" received at y is

$$\begin{aligned} M_1(y) &= \sum_j \mathrm{I}\{R_j > |X_j - y|\} = \int_{\mathbf{R}^d} \int_0^\infty \mathrm{I}\{r > |x - y|\}\, N(dx, dr) \\ &= \#\text{ balls containing } y \end{aligned}$$

If $E(R^d) < \infty$ then

$$\begin{aligned} EM_1 &= \int_{\mathbf{R}^d} \int_0^\infty \mathrm{I}\{r > |x|\} \, n(dx, dr) = \lambda \int_0^\infty \int_{\mathbf{R}^d} \mathrm{I}\{r > |x|\} \, dx \, F_R(dr) \\ &= \lambda \int_0^\infty |B(0, r)| \, F_R(dr) = \lambda |B(0, 1)| \, E(R^d). \end{aligned}$$

The moment generating function

$$\begin{aligned} \log E e^{\theta M_1} &= \int_{\mathbf{R}^d} \int_0^\infty e^{\theta \mathrm{I}\{r > |x|\}} - 1) \, n(dx, dr) \\ &= \lambda(e^\theta - 1) \int_{\mathbf{R}^d} \int_0^\infty \mathrm{I}\{r > |x|\} \, n(dx, dr) \\ &= \lambda(e^\theta - 1) |B(0, 1)| \, E(R^d) \end{aligned}$$

shows that M_1 is Poisson. For fixed $R = r$, a point is connected to at least one network node with probability

$$P(M_1 \geq 1) = 1 - e^{-\lambda |B(0,1)| r^d}.$$

Pathloss model

With each node associate signal of power S. Attenuation over distance x given by function $a(x)$, e.g.

$$a_0(x) = |x|^{-\beta}, \qquad a_1(x) = (1 + |x|)^{-\beta}, \quad \beta > d.$$

External noise W, threshold T, required signal to noise ratio:

$$\mathrm{SNR} = S\,a(x)/W > T.$$

The # of nodes successfully received at the origin:

$$M_2 = \sum_j \mathrm{I}\{S_j a(X_j) > TW\} = \int_{\mathbf{R}^d} \int_0^\infty \mathrm{I}\{s\,a(x) > TW\}\, N(dx, ds).$$

Pathloss model, cont'n

Using $a_0(x) = |x|^{-\beta}$,

$$M_2 = \int_{\mathbf{R}^d} \int_0^\infty \mathrm{I}\{(s/TW)^{1/\beta} > |x|\} \, N(dx, ds).$$

Thus, pathloss model equivalent to connectivity model with

$$R = (S/TW)^{1/\beta}, \quad F_R(r) = P(S < TWr^\beta) = F_S(TWr^\beta).$$

Hence

$$EM_2 = \lambda |B(0,1)| \, E[(S/TW)^{d/\beta}] = \lambda |B(0,1)| \, T^{-d/\beta} E(S^{d/\beta}) \, E(W^{-d/\beta}).$$

Basic assumption on S: $E(S^{d/\beta}) < \infty$. Since $\beta > d$, suffices to have $ES < \infty$. The additional moment condition for external noise is somewhat artificial; singularity of a_0.

Rayleigh fading

S exponential distribution, parameter μ. Motivation comes from underlying picture of the signal as a complex waveform $Z = X + iY$ with Gaussian real and imaginary parts. If X, Y independent zero mean Gaussian random variables with variance σ^2, then power of the wave is the squared amplitude $X^2 + Y^2$, which is exponential with mean $2\sigma^2$. In pathloss model:

$$P(R > r) = EP(S > TWr^\beta | N_0) = E(e^{-\mu TWr^\beta}), \quad r \geq 0.$$

Some Poisson integral calculus

Ref's: E.g. Kingman [Ki], Kallenberg [Ka].

Poisson point measure $N = \sum_j \delta_{X_j}$ defined on measurable state space $\mathbf{X}$. Intensity measure is a σ-finite measure n also defined on $\mathbf{X}$. For any $A \subset \mathbf{X}$, the number of points in A, $N(A) = \sum_j I\{X_j \in A\}$, is Poisson with mean $n(A)$. For $A_1, \ldots, A_n$ in $\mathbf{X}$ disjoint the variables $N(A_1), \ldots, N(A_n)$ are independent.

Let $f : \mathbf{X} \to \mathbf{R}$ be a measurable function. The stochastic integral of f with respect to N,

$$\int_{\mathbf{X}} f(x)\, N(dx) = \sum_j f(X_j),$$

exists with probability one if and only if

$$\int_{\mathbf{X}} \min(|f(x)|, 1)\, n(dx) < \infty.$$

For such f, the distribution of the Poisson integral is determined by the characteristic function

$$E \exp\Big\{ i\theta \int_{\mathbf{X}} f(x)\, N(dx) \Big\} = \exp\Big\{ \int_{\mathbf{X}} (e^{i\theta f(x)} - 1)\, n(dx) \Big\}, \quad \theta \in \mathbf{R}.$$

Poisson integral calculus, cont'n

In particular,

$$E \int_{\mathbf{X}} f(x)\, N(dx) = \int_{\mathbf{X}} f(x)\, n(dx),$$

$$\mathrm{Var} \int_{\mathbf{X}} f(x)\, N(dx) = \int_{\mathbf{X}} f(x)^2\, n(dx)$$

The centered stochastic integral

$$\int_{\mathbf{X}} f(x)\, N(dx) - E \int_{\mathbf{X}} f(x)\, N(dx) = \int_{\mathbf{X}} f(x)\, (N(dx) - n(dx))$$

with respect to the compensated measure $\widetilde{N}(dx) = N(dx) - n(dx)$, has characteristic function

$$E \exp\left\{ i\theta \int_{\mathbf{X}} f(x)\, \widetilde{N}(dx) \right\} = \exp\left\{ \int_{\mathbf{X}} (e^{i\theta f(x)} - 1 - i\theta f(x))\, n(dx) \right\}.$$

The integral $\int_{\mathbf{X}} f(x)\, \widetilde{N}(dx)$ exists in L^1 if and only if

$$\int_{\mathbf{X}} \min(|f(x)|, f(x)^2)\, n(dx) < \infty.$$

Multicast model

Users located in $\mathbf{R}^d$ as Poisson point process with intensity λdx. Signal of power S emitted at the origin. The users are potential receivers. Transmission subject to attenuation pathloss, $a_0(x) = |x|^{-\beta}$, and external noise W. The # of users that recieve the message is

$$M_3 = \sum_j \mathrm{I}\{S\, a(X_j) > TW\} = \int_{\mathbf{R}^d} \mathrm{I}\{S\, a(x) > TW\}\, N(dx).$$

Characteristic function:

$$\begin{aligned} E(e^{i\theta M_3}) &= E \exp\left\{\lambda(e^{i\theta} - 1) \int_{\mathbf{R}^d} \mathrm{I}\{S > WT|x|^\beta\}\, dx\right\} \\ &= E \exp\{\lambda(e^{i\theta} - 1)|B(0,1)|(S/WT)^{d/\beta}\}. \end{aligned}$$

Thus, M_3 is mixed Poisson random with random intensity that depends on non-fading signal to noise ratio S/W.

Interference model

The field of Poisson interference is the (stationary) shot noise process

$$I_\lambda(y) = \sum_j S_j a(X_j - y) = \int_{\mathbf{R}^d} \int_0^\infty s\, a(x-y)\, N(dx, ds), \quad y \in R^d.$$

For $I_\lambda = I_\lambda(0)$ with $a = a_0$ we have

$$\begin{aligned}
\log E(e^{i\theta I_\lambda}) &= \int_{\mathbf{R}^d} \int_0^\infty (e^{i\theta a(x) s} - 1)\, n(dx, ds) \\
&= \lambda |B(0,1)| \int_0^\infty E(e^{i\theta S/r^\beta} - 1) r^{d-1}\, dr \\
&= \lambda |B(0,1)| \int_0^\infty E(e^{i\theta S t} - 1) \beta^{-1} t^{-d/\beta - 1}\, dt \\
&= \lambda |B(0,1)| E(S^{d/\beta}) \int_0^\infty (e^{i\theta t} - 1) \beta^{-1} t^{-d/\beta - 1}\, dt \\
&= \lambda |B(0,1)| E(S^{d/\beta})\, C(\operatorname{sign}\theta)\, |\theta|^{d/\beta}.
\end{aligned}$$

Thus, I_λ is α-stable with stable index $\alpha = d/\beta < 1$ (infinite mean).

Interference model, cont'n

Place source of signal power S at $x \in \mathbf{R}^d$. Emitted signal is received at the origin uncorrupted by interference if signal to interference and noise ratio exceeds a threshold value

$$\mathsf{SINR} = \frac{S\,a(x)}{W + I_\lambda} > T.$$

Assuming Rayleigh fading with S exponential of mean $1/\mu$,

$$P(S\,a(x) > T(W + I_\lambda)) = E(e^{-\mu T W/a(x)})\,E(e^{-\mu T I_\lambda/a(x)}).$$

Here,

$$\begin{aligned} E(e^{-\mu T I_\lambda/a(x)}) &= \exp\left\{-\lambda C_{d,\beta} E(S^{d/\beta})\,(\mu T/a(x))^{d/\beta}\right\} \\ &= \exp\left\{-\lambda C_{d,\beta}\Gamma(1 + d/\beta)\,T^{d/\beta}|x|^d\right\} \\ &= \exp\left\{-\lambda \frac{d\pi/\beta}{\sin(d\pi/\beta)}\,T^{d/\beta}|x|^d\right\} \end{aligned}$$

Node density balancing interference

Medium access control probability, [BBM'06]. No external noise, $W = 0$. Assume each station which access the medium (prob p) expects to transmit over fixed distance r with threshold T, to a destination user not considered part of the network.
If accessing station is (X_j, S_j) and the user located at Y_j, $|X_j - Y_j| = r$, then success if $S_j\, a(X_j - Y_j) > T I_{\lambda p}(Y_j)$. Hence the expected # of successful users in $\mathbf{S} \subset \mathbf{R}^d$ equals

$$
\begin{aligned}
E \sum_{X_j \in \mathbf{S}} \mathrm{I}\{S_j a(r) > T I_{\lambda p}(Y_j)\} &= \int_{\mathbf{S}} P(Sa(r) > T I_{\lambda p})\, \lambda p dx \\
&= \lambda p |\mathbf{S}| P(Sa(r) > T I_{\lambda p}) \\
&= \lambda' p_r(\lambda')\, |\mathbf{S}|, \quad \lambda' = \lambda p.
\end{aligned}
$$

Thus, maximize $\lambda p_r(\lambda)$ over λ.

Node density versus interference, cont'n

Claim: If $ES^p < \infty$ for some $p > d/\beta$, then there exists an optimal node intensity $\lambda_{\max}$ which maximizes the performance of the network, under given conditions.
Chebyshev:

$$p_r(\lambda) = p_{\lambda^{1/d}r}(1) = P(S > T\lambda^{\beta/d}r^{\beta}I_1) \leq E(S^p)E(I_1^{-p})\frac{1}{T^p\lambda^{p\beta/d}r^{p\beta}}$$

Here,

$$\begin{aligned} E(I_1^{-p}) &= \frac{1}{\Gamma(p)}\int_0^\infty s^{p-1}E(e^{-I_1 s})\,ds = \cdots = \\ &= \frac{(\beta/d)\Gamma(p\beta/d)}{\Gamma(p)(|B(0,1)|\Gamma(1-d/\beta)E(S^{d/\beta})/d)^{p\beta/d}} < \infty. \end{aligned}$$

Thus, if $E(S^p) < \infty$, some $p > d/\beta$, then

$$\lambda p_r(\lambda) \leq \text{const}\frac{1}{\lambda^{p\beta/d-1}} \to 0, \quad \lambda \to \infty,$$

Traffic session modeling

Pitman-Yor (and others): There exists a two-parameter stochastic process $\{\Gamma_v(t), v \geq 0, t \geq 0\}$ which is a gamma subordinator process in t and a squared Bessel diffusion in v.
Interpretation: Subordinator increments yield the cumulative increase of energy pulses from a given emitter over time. For fixed t, $\Gamma_v(t), v \geq 0$, $\Gamma_0(t) = 0$, is a squared Bessel diffusion with fractal dimension $2t$ and variance parameter $v/2$. In particular,

$$\Gamma_v(k) = \sum_{j=1}^{k} (X_j^2 + Y_j^2), \quad (X_j, Y_j) \quad \text{zero mean Gaussian, variance } v/2,$$

meaning that Rayleigh fading stems from variations in squared amplitude of complex Gaussian wave.

Rayleigh fading sessions

Let $N_\lambda(dx, d\gamma)$ be Poisson point process in $R^d \times \mathcal{D}$ with intensity measure $\lambda dx\, Q_0^{a(x)}(d\gamma)$, where $Q_0^v(d\gamma)$ is the distribution for subordinator paths $\{\gamma(t), t \geq 0\}$ of $\Gamma_v(t)$.
The cumulative interference in y at time t is given by

$$I_\lambda(t, y) = \int_{\mathbf{R}^d} \int_{\mathcal{D}} \gamma(t) a(x - y)\, N_\lambda(dx, d\gamma).$$

Using $a = a_0$,

$$\begin{aligned} \log E(e^{i\theta I_\lambda(t)}) &= \int_{\mathbf{R}^d} \int_{\mathcal{D}} (e^{i\theta\gamma(t)} - 1)\, \lambda dx\, Q_0^{a(x)}(d\gamma) \\ &= \lambda |B(0,1)| E(\Gamma_1(t)^{d/\beta})\, C(\operatorname{sign}\theta)\, |\theta|^{d/\beta}. \end{aligned}$$

Lognormal fading

Multiplicative effect of wave shadowing. Changes slowly in comparison to Rayleigh fading.

Assume the power observed at the origin of an emitter in x has lognormal distibution V_x with distribution $F_x(dv)$ and $EV_x = a_1(x)$. Conditional on $V_x = v$, assume the cumulative power is $\Gamma_v(t)$, $t \geq 0$.

Relevant Poisson measure $N_\lambda(dx, dv, d\gamma)$ has intensity $\lambda dx\, F_x(dv)\, Q_0^v(d\gamma)$, and

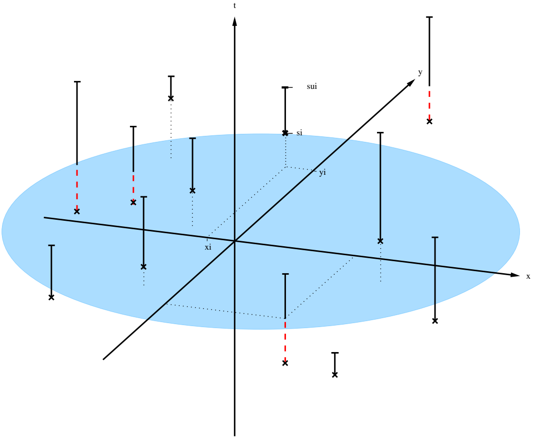
$$\begin{aligned}
\log E(e^{i\theta I_\lambda(t)}) &= \int_{\mathbf{R}^d} \int_0^\infty \int_{\mathcal{D}} (e^{i\theta\gamma(t)} - 1)\, \lambda dx\, F_x(dv) Q_0^v(d\gamma) \\
&= \lambda \int_{\mathbf{R}^d} \int_0^\infty E(e^{i\theta\Gamma_v(t)} - 1)\, F_x(dv) dx \\
&= \lambda \int_{\mathbf{R}^d} E\Big[\Big(\frac{V_x}{1 - i\theta V_x}\Big)^t - 1\Big]\, dx.
\end{aligned}$$

Temporal-Spatial Interference

Signal transmitters:

- random locations $x \in \mathbf{S} \subset \mathbf{R}^d$, Poisson
- initial times $s \in \mathbf{R}$, Poisson
- call holding times u, law $G(du)$

Transmission sessions (s, x, u), given by Poisson point measure $N(ds, dx, du)$ with intensity $\lambda ds dx\, G(du)$



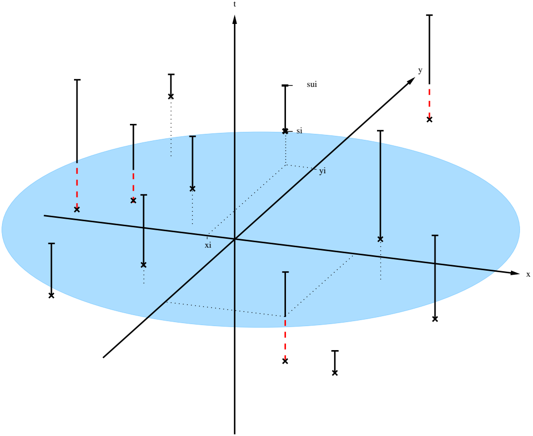
Temporal-Spatial Interference, cont

Interested in total spatial interference, measured as received power at origin. Two types of fading reduce signal power:

- Lognormal fading; multiplicative shadowing, long term
- Rayleigh fading; multipath interaction, short term

Model:

- Attenuation function, $g(x) = \frac{1}{(1+|x|)^{\beta}}$
- $V \sim \log N$, EV=g(x), law $F_x(dv)$
- Given $V = v$, power given by Gamma subordinator $\Gamma_v(t)$, law $Q^v(d\gamma)$



Temporal-Spatial Interference, cont

The resulting signal of session (s, x, u) is a point (s, x, u, v, γ) given by a Poisson point measure $N(dsdx, du, dv, d\gamma)$ with intensity measure $\lambda dsdx\, G(du)\, F_x(dv)\, Q^v(d\gamma)$.

Introduce

$$K_t(s, u) = \int_0^t I\{s < y < s + u\}\, dy = |(s, s + u) \cap (0, t)|,$$

which measures the fraction of the time interval $[0, t]$ during which a session that starts at time s and has duration u is active.

Interference process:

$$I_\lambda(t) = \int_{\mathbf{R}\times\mathbf{R}^d} \int_0^\infty \int_0^\infty \int_{\mathcal{D}} \gamma(K_t(s, u))\, N(dsdx, du, dv, d\gamma).$$

Fluctuations

Fluctuations of the Poisson interferers around the mean level

$$\begin{aligned} EI_\lambda(t) &= \int_{\mathbf{R}\times\mathbf{R}^d}\int_0^\infty\int_0^\infty E\Gamma_v(K_t(s,u))\,\lambda dsdx\,F_x(dv)\,G(du) \\ &= \int_{\mathbf{R}\times\mathbf{R}^d}\int_0^\infty E(V_x)K_t(s,u)\,\lambda dsdx\,F(du) \\ &= \lambda\int_{\mathbf{R}^d} EV_x\,dx\,\int_{-\infty}^\infty\int_0^\infty K_t(s,u)\,ds\,F(du) \\ &= \lambda\int_{\mathbf{R}^d} EV_x\,dx\;EU\;t, \end{aligned}$$

are described by the compensated Poisson integral

$$J_\lambda(t) = I_\lambda(t) - EI_\lambda(t) = \int_{\mathbf{R}\times\mathbf{R}^d}\int_0^\infty\int_0^\infty\int_{\mathcal{D}} \gamma(K_t(s,u))\,\widetilde{N}_\lambda(dsdx,du,dv,d\gamma),$$

where

$$\widetilde{N}_\lambda(dsdx,du,dv,d\gamma) = N_\lambda(dsdx,du,dv,d\gamma) - \lambda dsdx\,G(du)\,F_x(dv)\,Q_0^v(d\gamma).$$

Scaling Analysis

Investigate scaling limits of

$$\begin{aligned}
&\log E(e^{i\theta J_\lambda(t)}) \\
&\quad = \int_{\mathbf{R}\times\mathbf{R}^d} \int_0^\infty \int_0^\infty E(e^{i\theta\Gamma_v(K_t(s,u))} - 1 - i\theta\Gamma_v(K_t(s,u)))\, \lambda ds dx\, G(du) F_x(dv) \\
&\quad = \int_{\mathbf{R}\times\mathbf{R}^d} \int_0^\infty \int_0^\infty (e^{-K_t(s,u)\log(1-iv\theta)} - 1 - iv\theta K_t(s,u))\, \lambda ds dx\, G(du) F_x(dv)
\end{aligned}$$

We look at high-density limits, $\lambda \to \infty$, under time rescaling, $t \to at$, $a \to \infty$.

Finite variance call holding time

Suppose $E(U^2) < \infty$. Then

$$
\begin{aligned}
&\log E(e^{i\theta b^{-1} J_\lambda(at)}) \\
&\quad \sim -\frac{1}{2} \int_{\mathbf{R}\times\mathbf{R}^d} \int_0^\infty \int_0^\infty \left(\frac{v\theta}{b}\right)^2 (K_{at}(s,u)^2 + K_{at}(s,u))\, \lambda ds dx\, G(du) F_x(dv) \\
&\quad \sim -\frac{1}{2}\theta^2 \int_{\mathbf{R}^d} E(V_x^2)\, dx \int_{-\infty}^\infty \int_0^\infty (K_{at}(as,u)^2 + K_{at}(as,u))\, ds\, G(du) \\
&\quad \sim -\frac{1}{2}\theta^2 \int_{\mathbf{R}^d} E(V_x^2)\, dx\, (E(U^2) + E(U))\, t,
\end{aligned}
$$

since

$$
K_{at}(as,u) \to u\, \mathrm{I}\{0 < s < t\}, \quad a \to \infty.
$$

The distributional limit of $J_\lambda(at)/\sqrt{\lambda a}$ is Brownian motion with variance $\int_{\mathbf{R}^d} E(V_x^2)\, dx\, E(U^2 + U)$.

Scaling analysis, heavy tails

Assume that distribution $G(du)$ for call durations has a regularly varying tail at infinity, $1 - G(u) = L(u)u^{-\gamma}$, L a slowly varying function, γ, $1 < \gamma < 2$ the index of regular variation. Then U has finite mean but infinite variance.

Three possible scaling regimes given by relative speed at which λ and a tend to infinity. We consider

- Fast connection rate: $\lambda / a^{\gamma - 1} \to \infty$, $b^2 = \lambda a^{3-\gamma}$
- Intermediate connection rate: $\lambda / a^{\gamma - 1} \to 1$, $b = a$
- Slow connection rate: $\lambda / a^{\gamma - 1} \to 0$, $b^\gamma = \lambda a$

Put differently: While increasing the density of nodes, trace the system along appropriate time scale. Which fluctuations build up?

Scaling analysis, heavy tails, cont'n

One can show that in each of the three cases

$$\log E(e^{i\theta J_\lambda(at)/b})$$
$$\sim \int_{\mathbf{R}\times\mathbf{R}^d} \int_0^\infty \int_0^\infty (e^{iv\theta K_{at}(s,u)/b} - 1 - iv\theta K_{at}(s,u)/b)\, \lambda ds dx\, G(du) F_x(dv).$$

For fast connection rate

$$\begin{aligned}
&\log E(e^{i\theta J_\lambda(at)/b}) \\
&\qquad \sim -\frac{1}{2}\int_{\mathbf{R}^d} EV_x^2\, dx \int_{-\infty}^{\infty}\int_0^\infty (a\theta K_t(s,u)/b)^2\, \lambda a ds\, G(adu) \\
&\qquad \sim -\frac{1}{2}\theta^2 \int_{\mathbf{R}^d} EV_x^2\, dx \int_{-\infty}^{\infty}\int_0^\infty K_t(s,u)^2\, ds\, u^{-\gamma-1} du.
\end{aligned}$$

A Gaussian distribution. Which one?

Scaling analysis, heavy tails, cont'n

Here

$$\begin{aligned}\int_{-\infty}^{\infty}\int_0^{\infty} K_t(s,u)^2\,ds\,u^{-\gamma-1}du \\ &= \int_0^t\int_0^t dydy'\int_{|y-y'|}^{\infty}(1-|y-y'|/u)\,u^{-\gamma-1} \\ &= \mathrm{const}\int_0^t\int_0^t dydy'|y-y'|^{-(\gamma-1)} = \mathrm{const}\,t^{3-\gamma}.\end{aligned}$$

In general, we obtain the finite-dimensional distribution of fractional Brownian motion with Hurst index $H = (3-\gamma)/2$.

Scaling analysis, heavy tails, cont'n

Intermediate scaling yields

$$\log E(e^{i\theta J_\lambda(at)/a}) \\ \to \int_{\mathbf{R}\times\mathbf{R}^d}\int_0^\infty\int_0^\infty (e^{iv\theta K_t(s,u)} - 1 - iv\theta K_t(s,u))\, dsdx\, u^{-\gamma-1}F_x(dv),$$

which is the characteristic function of

$$Y_\lambda(t) = \int_{\mathbf{R}\times\mathbf{R}^d}\int_0^\infty\int_0^\infty K_t(s,u)\,\widetilde{N}(dsdx,du,dv), \quad t\geq 0,$$

where $\widetilde{N}$ is a compensated Poisson measure with intensity measure $dsdx\, u^{-\gamma-1}F_x(dv)$. The covariance structure of this process is same as that of FBM with Hurst index $3-\gamma$.

Finally, the limit in the case of slow connection rate is a stable Lévy process with stable index $1/\gamma$.