1 Notes on throughput in the wireless network model

Let Φ_{UE} and Φ_{BS} denote two independent Poisson point process in $\mathbb{R}^d$ with intensity measures $\lambda_0\, dx$ and $\lambda_1\, dx$, respectively. The points in Φ_{UE} represent the locations of user equipment and the points in Φ_{BS} the locations of base stations for a wireless network system.

For a given configuration of users and base stations we apply a time-slotted packet transmission model. In each time slot each user (UE) with probability p attempts to transmit a packet to the station (BS) which is nearest in Euclidean distance. Transmission attempts are independent between slots and between users. The signaling power for a user at x is given by an exponential random variable S_x with mean μ, subject to pathloss according to a given function a. A packet transmission is successful if the user is able to establish a connection with the base station such that the signal to interference ratio exceeds a preset threshold value T. The total interference at a point z in a fixed time slot is given by

$$I(z) = \int sa(x-z)\mathcal{N}_{\lambda_0 p}(dx, ds)$$

where $\mathcal{N}(dx, ds)$ is a Poisson measure on $\mathbb{R}^d \times \mathbb{R}_+$ with intensity $\lambda_0 p\, dx\, \mu^{-1} e^{-s/\mu}\, ds$.

We consider the Voronoi tesselation formed by the Poisson points Φ_{BS} as nodal center points for Voronoi cells $\{C_i, x_i \in \Phi_{\mathrm{BS}}\}$.

Let M be the number of successful packet transmissions in a fixed cell for which we may assume that the base station is located at $z = 0$.

Let's restrict to $d = 2$. By known results for Voronoi tessellations (Zuev, Foss 1995?), we have

$$E\Big[\sum_{x\in C} f(x)\Big] = \lambda_0 \int_{\mathbb{R}^2} f(x)\, e^{-\lambda_1 \pi |x|^2}\, dx.$$

Thus, the expected value of

$$M = \sum_{x \in C} 1_{\{S_x\, a(x-z) > T\cdot I(0)\}}$$

is given by

$$\begin{aligned}
\mathbb{E}(M) &= \mathbb{E}\Big[\sum_{x\in C} P(S > TI_0/a(x)|\cdot)\Big] = \mathbb{E}\Big[\sum_{x\in C} \mathbb{E}[e^{-TI(0)/\mu a(x)}|\cdot]\Big] \\
&= \lambda_0 p \int_{\mathbb{R}^2} \mathbb{E}[e^{-TI(0)/\mu a(x)}|\cdot]\, e^{-\lambda_1\pi|x|^2}\, dx.
\end{aligned}$$

Here,

$$\mathbb{E}[e^{-TI(0)/\mu a(x)}|\cdot] = \cdots = \exp\Big\{-p\int \frac{Ta(y)/a(x)}{1+Ta(y)/a(x)}\, dy\Big\}$$

Hence

$$\begin{aligned}
\mathbb{E}(M) &= \lambda_0 p \int_0^\infty \exp\Big\{-p\int \frac{Ta(y)/a(v)}{1+Ta(y)/a(v)}\, dy\Big\}\, e^{-\lambda_1\pi v^2}\, 2\pi v\, dv \\
&=
\end{aligned}$$

We are interested in $a_0(x) = |x|^{-\beta}$, $\beta > d$, and

$$a_\kappa(x) = \frac{\kappa}{\kappa + |x|^\beta}, \quad \beta > d.$$

Perhaps also $a(x) = e^{-\kappa|x|}$.

We expect $E(M)$ to behave as a function of the type $pe^{-constp}$, $0 \leq p \leq 1$, with near linear increase up to a maximum and then decaying efficiency.