UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Jörgen Östensson

Applied Mathematics
Fall 2012

Transform theory problems

1. Solve the initial value problem

$$x''(t) - x'(t) - 2x(t) = 2, \quad x(0) = x'(0) = 1.$$

2. Solve the integral equation

$$y(t) = t + \int_0^t \sin(t-\tau)y(\tau)\,d\tau, \quad t \geq 0.$$

3. Solve the problem

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0, \quad x > 0, \quad t > 0, \\ u(0,t) = a\sin\omega t, \quad t > 0, \\ u(x,0) = 0, \ u_t(x,0) = 0, \quad x > 0. \end{cases}$$

4. Determine the fouriertransform $\hat{f}(\omega)$ of the function $f(t) = te^{-|t|}$.

5. Find a function $u \in L^1(\mathbb{R})$ such that

$$u(t) + \int_{-\infty}^{\infty} e^{-|t-s|}u(s)\,ds = e^{-|t|}.$$

6. Solve the problem

$$\begin{cases} u_t - u_x - u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0, \\ u(x,0) = H(x+1) - H(x-1), \quad x \in \mathbb{R}. \end{cases}$$

(Here $H(x)$ denotes the Heaviside function, which is equal to 1 if x is positive and 0 otherwise.)

Answers or hints:

1. $x(t) = e^{2t} + e^{-t} - 1$.

2. $y(t) = t + \frac{t^3}{6}$.

3. $u(x,t) = a \sin\left(\omega(t - \frac{x}{c})\right) H(t - \frac{x}{c})$, where H denotes the Heaviside function.

4. $\hat{f}(\omega) = -\frac{4iw}{(1+\omega^2)^2}$.

5. $u(t) = \frac{1}{\sqrt{3}} e^{-\sqrt{3}|t|}$.

6. $u(x,t) = \frac{1}{2}\left[\text{erf}\left(\frac{x+t+1}{2\sqrt{t}}\right) - \text{erf}\left(\frac{x+t-1}{2\sqrt{t}}\right)\right]$, where $\text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy$.