UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Jörgen Östensson

Functional Analysis I, 5 hp
Spring 2010

Functional Analysis I

Course literature

Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John-Wiley & Sons.

Some additional notes on the spectral theorem of compact self-adjoint operators will be handed out in class.

Course web page: http://www.math.uu.se/staff/pages/?uname=ostensson

Here you can find information about the course, e.g. materials handed out in class.

Teaching

Teaching consists of 16 lectures. These lectures will mainly be devoted to the basic theory. You are strongly advised to solve additional problems on your own.

The course material is shown under the heading ”Sections in book” in the lecture plan below. I will certainly not have time to cover everything in class, during which I can only hope to explain basic ideas and the most fundamental results. You are expected to study the rest on your own.

Preliminary lecture plan

Lecture	Contents	Sections in book
1	Metric spaces and their topology.	1.1–1.3
2	Convergence, Cauchy sequences and completeness.	1.3–1.5
3	Completion of metric spaces.	1.5–1.6
4	Vector spaces, normed spaces and Banach spaces.	2.1–2.3
5	Finite-dimensional normed spaces. Compactness.	2.4–2.5
6	Linear operators and functionals.	2.6–2.8
7	Normed spaces of operators. Dual spaces.	2.8–2.10
8	Inner product spaces. Hilbert spaces. Orthogonal projections.	3.1–3.3
9	Orthonormal sets and sequences. Totality.	3.3–3.6
10	Functionals on Hilbert space. The Hilbert-adjoint.	3.8–3.10
11	The Hahn-Banach theorem.	4.1–4.3
12	Reflexive spaces. The category theorem and the uniform boundedness theorem.	4.6 4.7
13	Strong and weak convergence. Convergence of sequences of operators.	4.8 4.9
14	The open mapping theorem. Closed linear operators. The closed graph theorem.	4.12 4.13
15	Spectral theory in normed spaces. Compact operators.	7.1–7.2, 8.1
16	The spectral theorem for compact self-adjoint operators.	Handout

Homework assignments

During the course I will hand out two homework assignments. The homework assignments are voluntary, but you are strongly encouraged to do them. Solving them leads to additional points on the exam in March, see below. Each assignment gives a maximum of 2 points, leading to a maximum of four additional points on the exam in March. Both assignments are evaluated with a score of 0, 1/2, 1, 3/2 or 2 points. The total score obtained raised to nearest integer will be added to the score on the exam in March. Note that the results from the homework problems will only affect the score on the exam in March.

The homework problems should be solved individually and handed in before deadline. You may either give them to me during class or leave them in my mailbox.

Examination

The course finishes on the 8th of March with a written exam. Maximum score: 40 points.

The results from homework assignments will be added to the score obtained on the exam. A total score of 18 is needed for the grade 3, 25 for the grade 4, and 32 for the grade 5.

From the course syllabus:

Learning outcomes

In order to pass the course (grade 3) the student should be able to

- use basic functional analytic formalism in normed spaces;
- use ON-systems and orthogonal projections in Hilbert spaces;
- solve simple problems about the weak topology;
- solve simple Hilbert space spectral theory problems.

Contents

Topology in metric spaces. Normed spaces. Banach spaces, inner product spaces, Hilbert spaces. Linear maps. Basic functional analytic theorems: Hahn-Banach's theorem, Banach-Steinhaus' theorem, the open map and the closed graph theorems. Weak convergence. Operators on Hilbert spaces. Geometry in Hilbert spaces. The spectral theory for compact symmetric operators.

Uppsala, 14th of January 2010.

Jörgen Östensson