**UPPSALA UNIVERSITET** 2005-10-06
Matematiska institutionen Matematik för signalbehandling 1IT080
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# Trial Exam with Comments

**GENERAL COMMENT: Time allocated for the real exam is 5 hours. Apart from writing materials you will be allowed to use a calculator and the *BETA*-textbook. (We will discuss the alternative - both textbooks in place of *BETA*.) All problems have relatively short solutions, so if your solution becomes very long it means that you are probably doing something unnecessary. The problems in the exam paper will be modelled on the problems and examples from both textbooks. The relationship of consecutive problems to the material in the textbooks is indicated below. Try to solve the problems from this trial exam at home. If you want your work to be corrected by me, hand it to me on Tuesday, October 11. We will discuss solutions in class in a week or so. You don't have to solve the last question as we have not covered the relevant material yet.**

**1.** Assume that $X_1$ and $X_2$ are independent random variables with the probability density functions given, respectively, by the formulas

$$f_{X_1}(x) = \begin{cases} \frac{2-|x-1|}{4} & \text{if } \; -1 \le x \le 3, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f_{X_2}(x) = \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{x^2}{8}\right).$$

Calculate the probability that both $X_1$ and $X_2$ are greater than 0.

**Comment about Question 1:** **This is a general question on probability and will correspond to the material from Sections: 2.9, 3.6-3.8, 4.8-4.11, 5.1-5.7 in the probability textbook.**

**2.** Show that if $X$ is a stationary stochastic process, then also the stochastic process $Y = \exp(X)$ must be stationary.

**Comment about Question 2:** **This question will correspond to the material from Chapter 10 in the probability textbook.**

**3.** Let

$$h_n = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

be the unit impulse response of a linear time invariant digital filter. Assume that the input of this filter is formed by a wide sense stationary sequence of random variables $X_n$ and that the output sequence $Y_n$ is also wide sense stationary. The autocorrelation function of the output is known to be

$$R_Y[n] = \begin{cases} 4 - 2|n| & \text{if } |n| \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Assuming that $R_X[n] = 0$ if $|n| \geq 2$, find the values of $R_X[-1], R_X[0]$ and $R_X[1]$.

**Comment about Question 3:** **This question will correspond to the material from Chapter 11 in the probability textbook. A typical other type of a question would be a continuous time version of the above problem involving Fourier transforms. Section 11.4 from the complex analysis textbook can be helpful here.**

**4.** Find all entire functions whose real part is given by $x^2 - y^2 + y + 1$.

**Comment about Question 4:** **This will be a general question in complex analysis and will correspond to the material from Chapters 2, 3, 5 as well as Section 8.8 in the complex analysis textbook.**

**5.** Calculate the contour integral

$$\int_{C_1(i)} \frac{\exp(iz)}{z^2 + 1} dz,$$

where the circle $C_1(i)$ is assumed to be positively oriented.

**Comment about Question 5:** **This question will correspond to the material from Chapter 6 and Sections 8.1-8.4 in the complex analysis textbook.**

**6.** Find the Laurent series expansion of the function

$$f(z) = \frac{2}{z(z+1)(z+2)}$$

in the annulus $A(0, 1, 2)$.

**Comment about Question 6:** **This question will be about Z-tranforms. This topic will be covered in my lectures but is not covered in the complex analysis textbook. On the other hand much of it is equivalent to the material from Chapter 7 about Taylor and Laurent series expansions.**

**7.** Find the image of the strip

$$\{z \in \mathbb{C} : 0 < \operatorname{Re} z < 1\}$$

through the Möbius transformation

$$T(z) = \frac{z-1}{z-2}.$$

**Comment about Question 7:** **This question will correspond to the material from Sections 9.1-9.3 in the complex analysis textbook.**

**8.** Find the Fourier series expansion of the odd function $U$ of period $2\pi$ such that $U(t) = \pi/2 - |t - \pi/2|$ if $0 \le t \le \pi$.

**Comment about Question 8:** **This question will correspond to the material from Chapter 11 in the complex analysis textbook, except for Section 11.4 which corresponds to Question 3 above.**