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1IT080
Informationsteknologi IT3
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Matematik för signalbehandling

Skrivtid: 15–20.

Tillåtna hjälpmedel: Writing materials, a calculator, the *BETA*-textbook.

LYCKA TILL!

1. The joint PDF of random variables X and Y is the function

$$f_{X,Y}(x,y) = \begin{cases} 3/4 & , \quad 0 \le y \le 1 - x^2, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Calculate the conditional expected value $E[X^2 | Y = 1/2]$.

2. Let X_n be a sequence of independent identically distributed random variables such that $E[X_m] = 1$ and $\mathrm{Var}[X_m] = 4$ for all m. Find the autocorrelation function $R_Y[m,k]$ and the autocovariance function $C_Y[m,k]$ for the sequence $Y_m = 2X_m - X_{m-1}$.

3. Suppose that a wide sense stationary random sequence X_n with $\mu_X = 1$ and with the autocorrelation function

$$R_X[n] = \begin{cases} 2 & \text{if } n = 0 \text{ or } n = \pm 1, \\ 0 & \text{otherwise,} \end{cases}$$

is the input of a linear time invariant filter with the unit impulse response $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$. Calculate the autocorrelation function $R_Y[n]$ of the output Y_n of this filter.

4. Show that all zeros of the polynomial $P(z) = z^5 - z + 16$ are in the disk $|z| < 2$.

5. Calculate the contour integral

$$\int_{C_{\sqrt{2}}(1+i)} \frac{dz}{(z-1)^2(1+z^2)},$$

where the circle $C_{\sqrt{2}}(1+i)$ (with center at $1+i$ and radius $\sqrt{2}$) is assumed to be positively oriented.

6. Find the discrete time signal $x[n]$ whose bilateral Z-transform is the function

$$X(z) = \frac{4}{(2z-1)(z+3)}$$

with the region of convergence $\mathrm{ROC} = \{z \in \mathbb{C} \,:\, 1/2 < |z| < 3\}$.

7. Find the image of the strip

$$\{z \in \mathbb{C} \,:\, 0 < \operatorname{Im} z < 1\}$$

through the Möbius transformation

$$T(z) = \frac{z-3}{z-1}.$$

8. A causal signal $x(t)$ is a solution of the following initial value problem:

$$x''(t) + 2x'(t) + 5x(t) = 4e^{-t}, \qquad x(0) = 1,\ x'(0) = 1.$$

Find the (unilateral) Laplace transform $X(s)$ of this signal.

GOOD LUCK!