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# Matematik för signalbehandling

**Skrivtid: 8–13.**

**Tillåtna hjälpmedel:** Writing materials, a calculator, the *BETA*-textbook.

**1.** Let $\lambda > 0$ be a constant. Assume that the joint PDF of random variables $X$ and $Y$ is the function

$$f_{X,Y}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & , \quad 0 \leq x \leq y, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find the conditional PDFs $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.

**2.** Let $X(t)$ be a wide sense stationary stochastic process for which $\mu_X = 2$ and whose average power is equal to 1. Let $Y$ be a random variable independent from $X(t)$ for all $t$. Assume also that $Y$ has the uniform distribution over the interval $[0, 2\pi]$, that is the PDF of $Y$ is

$$f_Y(y) = \begin{cases} 1/2\pi, & \text{if } 0 \leq y \leq 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\mathrm{Var}\,[X(t)]$ and $\mathrm{E}\,[X^2(t)\cos(2\pi t + Y)]$.

**3.** A discrete time stochastic signal $X[n]$ is assumed to be wide sense stationary with zero expected value and the correlation function

$$R_X[n] = \frac{1}{4}\delta[n+1] + \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1], \qquad n = 0, \pm 1, \pm 2, \ldots.$$

Show that the corresponding power spectral density function can take any value in the interval $[0, 1]$.

**4.** Show that the function $f(z) = z^5 + 15z + 1$ has exactly 4 zeros in the annulus $A = \{z \in \mathbb{C} : 3/2 < |z| < 2\}$.

**5.** Using residues calculate the integral

$$\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 4} dx.$$

**6.** Find the discrete time signal $x[n]$ whose bilateral Z-transform is the function

$$X(z) = \frac{2}{(z-1)(4-z)}$$

with the region of convergence $\mathrm{ROC} = \{z \in \mathbb{C} : 1 < |z| < 4\}$.

**7.** Find the image of the strip

$$\{z \in \mathbb{C} : -\pi < \operatorname{Im} z < 0\}$$

through the transformation

$$f(z) = \frac{e^z - i}{e^z + i}.$$

**8.** Using the (unilateral) Laplace transform find the causal signal $x(t)$ which is a solution of the following initial value problem:

$$x''(t) + x'(t) - 2x(t) = 0, \qquad x(0) = 1, \ x'(0) = 4.$$

**GOOD LUCK!**