

Matematik för signalbehandling

Skrivtid: 09.00–14.00.

Tillåtna hjälpmedel: Writing materials, a calculator, the *BETA*-textbook.

LYCKA TILL!

1. Suppose that X and Y are two random variables whose joint probability density function is

$$f_{X,Y}(x,y) = \begin{cases} \frac{3}{8}(1-x^2) & \text{if } |x| \leq 1 \text{ and } |y| \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the conditional variance $\text{Var}[X | Y = 1/3]$.

2. Let (X_n) be a sequence of independent identically distributed random variables such that $E[X_n] = 2$ and $\text{Var}[X_n] = 5$. The sequence serves as the input of a linear time invariant filter with the unit impulse response

$$h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2]).$$

Find the autocorrelation function R_Y for the output sequence (Y_n) .

3. Recall that

$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \leq 1/2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the convolution $\Lambda(t) = \text{rect}(t) * \text{rect}(t)$. What is the Fourier transform of the function Λ ?

4. Show that all zeros of the analytic function $f(z) = 3z^3 - 2z^2 - z - 7$ are in the annulus $A = \{z \in \mathbb{C} : 1 < |z| < 2\}$.

5. Use residues to calculate

$$\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 1} dx.$$

6. Find the inverse Z-transform of

$$X(z) = \frac{3}{2z^2 + iz + 1}, \quad |z| > 1.$$

7. Find the image of the horizontal strip

$$S = \{z \in \mathbb{C} : \pi < \text{Im } z < 2\pi\}$$

under the mapping

$$T(z) = \frac{ie^z + 2}{e^z - i}.$$

8. Find the Laplace transform of the causal signal $x(t)$ which solves the following initial value problem:

$$x''(t) + x'(t) - 2x(t) = 0, \quad x(0) = 2, \quad x'(0) = 3.$$

GOOD LUCK!