

## Matematik för signalbehandling

**Skrivtid: 09.00–14.00.**

**Tillåtna hjälpmedel:** Writing materials, a calculator, the *BETA*-textbook.

**LYCKA TILL!**

1. Let

$$f_{X,Y}(x,y) = \begin{cases} 9e^{-3y} & \text{if } 0 \leq x < y, \\ 0 & \text{otherwise} \end{cases}$$

be the joint probability density function of two random variables  $X$  and  $Y$ . Calculate the conditional expectation  $E[X | Y]$ .

2. Consider the stochastic process  $X(t) = t - S$ , where  $S$  is a random variable with the probability density function

$$f_S(s) = \begin{cases} e^{-s} & \text{if } s \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find the autocorrelation function  $C_X(t, \tau)$  and the autocovariance function  $R_X(t, \tau)$ .

3. Suppose that an independent identically distributed sequence  $X_n$ , where  $E[X_n] = 0$ ,  $\text{Var}[X_n] = 9$ , serves as the input of the averaging filter with the unit impulse response

$$h_n = \begin{cases} 1/3 & \text{for } n = 0, 1, 2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the power spectral density of the output.

4. Show that the analytic function  $g(z) = e^{z-2} - z^5$  has 5 zeros in the unit disc  $D_1(0) = \{z \in \mathbb{C} : |z| < 1\}$ .

5. Use the residue theorem to calculate

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2(x^2 + 4)}.$$

6. Find the inverse Z-transform of the function

$$X(z) = \frac{1}{z(z-i)^2}, \quad 0 < |z| < 1.$$

7. Find the image of the upper half of the unit disc

$$D_+ = \{z \in \mathbb{C} : \text{Im } z > 0 \text{ and } |z| < 1\}$$

under the mapping

$$T(z) = \left( \frac{1+z}{1-z} \right)^2.$$

8. Find the Laplace transform of the causal signal  $x(t)$  which satisfies the following initial value problem:

$$x''(t) + 2x'(t) + x(t) = t^4, \quad x(0) = 1, \quad x'(0) = 2.$$

**GOOD LUCK!**