FORMULA SHEET

1. The characteristic equations for the 1st order non-linear PDE $F(x, y, u, u_x, u_y) = 0$, in the notation from the book and lectures, are:

$$\frac{dx}{dt} = F_p, \quad \frac{dy}{dt} = F_q, \quad \frac{dz}{dt} = pF_p + qF_q,$$
$$\frac{dp}{dt} = -F_x - F_z p, \quad \frac{dq}{dt} = -F_y - F_z q,$$

and with $x(s, 0) = f(s)$, $y(s, 0) = g(s)$, $z(s, 0) = h(s)$, and $p(s, 0) = \phi(s)$, $q(s, 0) = \psi(s)$, the strip condition reads:

$$h'(s) = \phi(s)f'(s) + \psi(s)g'(s).$$

2. The characteristic eqs. for the 2nd order PDE with principal part $au_{xx} + bu_{xy} + cu_{yy}$ are:

$$\frac{dy}{dx} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$

3. The wave equation:
   - d'Alembert's formula:

$$u(x, t) = \frac{1}{2}\left(g(x + ct) + g(x - ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} h(\xi)\, d\xi.$$

   - Kirchhoff's formula:

$$u(x, t) = \frac{1}{4\pi}\frac{\partial}{\partial t}\left(t\int_{|\xi|=1} g(x + ct\xi)\, dS_\xi\right) + \frac{t}{4\pi}\int_{|\xi|=1} h(x + ct\xi)\, dS_\xi.$$

4. The Laplace equation:
   A fundamental solution $E(x)$ for the Laplace operator in $\mathbb{R}^n$ is given by:

$$E(x) = \begin{cases} \dfrac{1}{2\pi}\log|x| & , n = 2, \\ \dfrac{1}{(2-n)\omega_n}|x|^{2-n} & , n \geq 3. \end{cases}$$