Ergodic Theorems for Iterated Function Systems Controlled by Stochastic Sequences

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Abstract

This thesis concerns certain long run (ergodic) properties of stochastic sequences arising from random iteration of functions. In each iteration step a random variable from a specified stochastic controlling sequence is used to determine which function to iterate among functions from an iterated function system with in general an arbitrary number of mappings.

Weak distributional ergodic theorems and theorems of law of large numbers type are proved under average contractivity and stochastic boundedness assumptions.

Papers [A] and [B], which are related, treat random iteration of functions where the function to iterate in each step depends on the previous choices (controlling semi-Markov chain and regenerative sequence, respectively).

Paper [C] concerns random (i.i.d.) iteration of functions, and some applications within the ergodic theory of Markov chains. A (weak) distributional ergodic theorem including rates of convergence in the Kantorovich distance is proved.

Papers [D] and [E] are closely related and treat independent (asymptotically i.i.d.) iteration of functions, i.e. non-homogeneous (asymptotically

homogeneous) Markov chains. In paper [D], distributional ergodic theorems are proved for iterated function systems with compact state space and a countable number of functions satisfying stability conditions. The results in Paper [E] generalize the distributional ergodic theorem of Paper [C]. As a consequence, information on how asymptotically vanishing perturbations influence the convergence rate is obtained.

The thesis consists of an introduction and the following papers:

[A] Stenflo, Ö. (1996) Iterated Function Systems Controlled by a Semi-Markov Chain, *Theory Stoch. Process.*, **2**(**18**) (1-2), 305–313.

[B] Silvestrov, D. S. and Stenflo, Ö. (1998) Ergodic Theorems for Iterated Function Systems Controlled by Regenerative Sequences, *Research reports No 11, 1996*, Dept. of Mathematics, Umeå University. (To appear in *J. Theoret. Probab.*, **11** (3), 1998.)

[C] Stenflo, Ö. (1998) Ergodic Theorems for Markov Chains represented by Iterated Function Systems, *Research reports No 2, 1998*, Dept. of Mathematics, Umeå University.

[D] Stenflo, Ö. (1997) Ergodic Theorems for Iterated Function Systems with Time Dependent Probabilities, *Research reports No 18, 1997*, Dept. of Mathematics, Umeå University. (To appear in *Theory Stoch. Process.*)

[E] Stenflo, Ö. (1998) Ergodic Theorems for Time-Dependent Random Iteration of Functions, *Research reports No 9, 1998*, Dept. of Mathematics, Umeå University. (Accepted for presentation at Fractal 98, 25–28 Oct. 1998, Malta.)

Key words: Ergodic theorems, Iterated Function Systems (IFS), Limit theorems, Convergence rates, Markov chains.

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