2010-08-24

Advanced Level Course, Fall 2010:

APPLIED LOGIC (10 hp)
- for computation and verification

Methods of mathematical logic are becoming widely used in engineering of computer systems and programs in order to guarantee correctness of operation and simplify construction and modification. The formal specification and proof of correctness of systems require different logical languages such as modal logics, equational logic, predicate calculus or type theory, and often employ automatic tools: theorem provers, model checkers, proof support systems and methods for program extraction and synthesis. To build, and use, such tools reliably one need to understand the mathematics behind them — mathematical logic — to a larger or lesser extent. The aim of this course is to present some of the most accessible and relevant parts of mathematical logic for such applications. These include proof-theoretic methods and algorithmtic aspects of logic.

During the course interactive theorem provers will be used, mainly the system Coq.

Contents

Introduction to the proof assistant Coq and review of basic logic. Natural deduction. Contexts. Proof-objects. Terms and type checking.

Constructive logic, type theory and functional programs: lambda calculus, simple type theory, intuitionistic logic, Martin-Löf type theory, propositions-as-types, principles of program extraction from proofs.

The system Coq: goals and proof tactics. Theorems and definitions. Equality and rewriting. Inductive data types and predicates.

Equational logic: terms, unification, universal algebra, equational reasoning, term rewriting.

Predicate logic and proof search: the completeness theorem, proof search in some calculi (tableaux, resolution). Solvable and unsolvable problems: complete and decidable theories, quantifier elimination, Gödel's incompleteness theorem (without proof).

Propositional logic: combinatorial problems as propositional problems. Methods for efficient solution and representation of propositional problems (Davis–Putnam, BDDs).

Modal logic: possible worlds semantics, Kripke models. Interpretations of modal logic: Temporal logic and epistemic logic. Applications in model checking.

Literature

M R A Huth and M F Ryan. *Logic in Computer Science: Modelling and reasoning about systems.* Second edition. Cambridge University Press 2004.

E Palmgren. *Constructive Logic and Type Theory* Course Notes, Uppsala 2004.

Supplementary material on decidability, equational logic, modal logic, term rewriting and intuitionistic logic.

Documentation and tutorials on the Coq system.

Level and prerequisites. The course is at the advanced level in mathematics. Prerequisites are B.Sc., Logic and Proof Techniques I, Automata Theory

Course start: Friday August 31, 13.15 – 15.00, in room 2:214, MIC (Polacksbacken), Uppsala University.

Examination: Assignments including laboratory work with computers (70% of total score). A short written examination at the end of the course (30 % of total score).

Teacher

Professor Erik Palmgren, tel. 018-471 32 85, Email: `palmgren@math.uu.se`.
Web page of the course: `www.math.uu.se/~palmgren/tillog`.

Responsible department: Department of Mathematics.