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Assignment 6
APPLIED LOGIC, Fall-10
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Assignment 6

Basic modal logic and model checking

The problems should be solved individually and are due by December 17. (If you hand in solutions earlier, please do this by email. Scan work written by hand.)

1. Let P and Q be propositional variables. Decide which of the following formulas that are provable in intutionionstic propositional logic. Give a proof, or a model where it is false. You can use Heyting algebra semantics or Kripke models.

   (a) $(P \to Q) \vee (Q \to P)$

   (b) $\neg(P \vee Q) \to \neg P \wedge \neg Q$

   (c) $\neg P \wedge \neg Q \to \neg(P \vee Q)$

   (d) $(P \vee \neg P) \to \neg\neg P$

   (e) $\neg\neg P \to (P \vee \neg P)$

   (5p)

2. Exercises 3.4.8 in Huth and Ryan (2004). (5p)

3. Exercises 3.4.10 (e) – (g) in Huth and Ryan (2004). (4p)

4. Exercises 3.5.1 in Huth and Ryan (2004). (5p)

5. Suppose that $\mathcal{M}$ is a model of basic modal logic . Let A be an arbitrary basic modal formula. Prove, or find references for the following results in the text book.

   (a) If R is reflexive, then $\mathcal{M} \models \Box A \to A$.

   (b) If R is transitive, then $\mathcal{M} \models \Box A \to \Box\Box A$.

   (c) If R is symmetric, then $\mathcal{M} \models \Diamond\Box A \to A$.

(d)* If R is a linear order, then $\mathcal{M} \models \Diamond\Box\Diamond A \leftrightarrow \Box\Diamond A$ (2 p)

6. Let $\mathcal{M}$ be a model for basic modal logic where the accessibility relation is a linear order. Let $u \in \{\Box, \Diamond\}^*$ denote an arbitrary string of modal operators. If A is a basic modal formula, then $u\,A$ is a modal formula. Show using Exercise 5 that $u\,A$ is is semantically equivalent to one of the formulae

$$A \qquad \Box A \qquad \Diamond A \qquad \Box\Diamond A \qquad \Diamond\Box A.$$

(4p)
