æ FORMELSAMLING
A: Trigonometriska formler

$$\tan x = \frac{\sin x}{\cos x} \tag{A1}$$

$$\cot x = \frac{\cos x}{\sin x} \tag{A2}$$

$$\sin(-x) = -\sin x \tag{A3}$$

$$\cos(-x) = \cos x \tag{A4}$$

$$\cos(\pi \pm x) = -\cos x \tag{A5}$$

$$\sin(\pi - x) = \sin x \tag{A6}$$

$$\tan(\pi + x) = \tan x \tag{A7}$$

$$\tan(\pi - x) = -\tan x \tag{A8}$$

$$\sin(\frac{\pi}{2} - x) = \cos x \tag{A9}$$

$$\cos(\frac{\pi}{2} - x) = \sin x \tag{A10}$$

$$\sin^2 x + \cos^2 x = 1 \tag{A11}$$

$$\sin 2x = 2\sin x \cos x \tag{A12}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \tag{A13}$$

$$\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x) \tag{A14}$$

$$\cos^2 \frac{x}{2} = \frac{1}{2}(1 + \cos x) \tag{A15}$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x} \tag{A16}$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x \tag{A17}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin y \sin x \tag{A18}$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y} \tag{A19}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \tag{A20}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2} \tag{A21}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \tag{A22}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2} \tag{A23}$$

radianer:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
grader:	0	30	45	60	90
sin:	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos:	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan:	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ej def.

Några extra formler att tänka på.

Linjens parameterform i rymden: $(x\,,\,y\,,\,z) = (x_0 + t\cdot r_1\,,\,y_0 + t\cdot r_2\,,\,z_0 + t\cdot r_3)$

eller i koordinatform $\begin{cases} x = x_0 + t\cdot r_1 \\ y = y_0 + t\cdot r_2 \qquad t\in\mathbf{R}\,. \\ z = z_0 + t\cdot r_3 \end{cases}$

Planets ekvation i parameterfri form: $ax + by + cz + d = 0$

eller $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0\,.$

Vinkelräta projektionen av en vektor $\mathbf{u}$ i den **normerade** riktningen $\mathbf{r}$:

$$\text{proj}_{\mathbf{r}}\mathbf{u} = (\mathbf{u}\circ\mathbf{r})\cdot\mathbf{r}\,.$$

Potenslagarna		Logaritmlagarna	
I.	$a^b\cdot a^c = a^{b+c}$	I.	$\log_a(c\cdot d) = \log_a c + \log_a d$
II.	$\frac{a^b}{a^c} = a^{b-c}$	II.	$\log_a\frac{c}{d} = \log_a c - \log_a d$
III.	$(a^b)^c = a^{bc}$	III.	$\log_a d^c = c\cdot\log_a d$

B: Standardgränsvärden

$$\lim_{x\to\infty} \frac{1}{x} = 0 \tag{B1}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \tag{B2}$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1 \tag{B3}$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1 \tag{B4}$$

$$\lim_{x\to\infty} \frac{x^p}{e^x} = 0 \tag{B5}$$

$$\lim_{x\to\infty} \frac{\ln x}{e^x} = 0 \tag{B6}$$

$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0 \tag{B7}$$

$$\lim_{x\to\infty} \left(1 + \frac{b}{x}\right)^x = e^b \tag{B8}$$

$$\lim_{n\to\infty} \frac{a^n}{n!} = 0 \tag{B9}$$

$$\lim_{n\to\infty} \sqrt[n]{a} = 1 \tag{B10}$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1 \tag{B11}$$

$$\lim_{x\to 0} x \ln|x| = 0 \tag{B12}$$

C: Standardderivator

$$Dx^a = ax^{a-1} \tag{C1}$$

$$D\sin x = \cos x \tag{C2}$$

$$D\cos x = -\sin x \tag{C3}$$

$$D\tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x \tag{C4}$$

$$De^x = e^x \tag{C5}$$

$$D\ln|x| = \frac{1}{x} \tag{C6}$$

$$Da^x = a^x \ln a\,, \quad a > 0 \tag{C7}$$

$$D\arcsin x = \frac{1}{\sqrt{1-x^2}} \tag{C8}$$

$$D\arctan x = \frac{1}{1+x^2} \tag{C9}$$

$$D\sqrt{x} = \frac{1}{2\sqrt{x}} \tag{C10}$$

$$D\arccos x = -\frac{1}{\sqrt{1-x^2}} \tag{C11}$$

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

$$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$$

D: Standardintegraler

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C\,, \quad a \neq -1 \tag{D1}$$

$$\int \sin x\,dx = -\cos x + C \tag{D2}$$

$$\int \cos x\,dx = \sin x + C \tag{D3}$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C \tag{D4}$$

$$\int \frac{1}{\sin^2 x}\,dx = -\frac{1}{\tan x} + C \tag{D5}$$

$$\int e^x\,dx = e^x + C \tag{D6}$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \tag{D7}$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C \tag{D8}$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C \tag{D9}$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \ln(x + \sqrt{1+x^2}) + C \tag{D10}$$

Om $F'(x) = f(x)$ då:

$$\int f(x)g(x)\,dx = F(x)\cdot g(x) - \int F(x)\cdot g'(x)\,dx$$

$$\int f\cdot g = F\cdot g - \int F\cdot g'$$

$$\int f(g(x))\cdot g'(x)\,dx = \int f(t)\,dt = F(t) + C = F(g(x)) + C\,.$$

E: Maclaurinutvecklingar

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + x^{n+1}A(x)$$
$$= \sum_{k=0}^{n} \frac{x^k}{k!} + x^{n+1}A(x) \qquad \textbf{(E1)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + x^{2n+3}A(x)$$
$$= \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + x^{2n+3}A(x) \qquad \textbf{(E2)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + x^{2n+2}A(x)$$
$$= \sum_{k=0}^{n} (-1)^n \frac{x^{2k}}{(2k)!} + x^{2k+2}A(x) \qquad \textbf{(E3)}$$

$$\tan x = x + \frac{x^3}{3} + x^5 A(x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^{n-1} \frac{x^n}{n} + x^{n+1}A(x)$$
$$= \sum_{k=1}^{n} (-1)^{k-1} \frac{x^k}{k} + x^{n+1}A(x) \qquad \textbf{(E4)}$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + (-1)^n \frac{x^{2n+1}}{2n+1} + x^{2n+3}A(x)$$
$$= \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + x^{2n+3}A(x) \qquad \textbf{(E5)}$$

$$\arcsin x = x + \frac{x^3}{6} + x^5 A(x) \qquad \textbf{(E6)}$$

$$(1+x)^a = 1 + \tfrac{a}{1!}x + \tfrac{a(a-1)}{2!}x^2 + \tfrac{a(a-1)(a-2)}{3!}x^3 + \ldots + \tfrac{a(a-1)\cdots(a-n+1)}{n!}x^n + x^{n+1}A(x)$$
$$= \sum_{k=0}^{n} \tbinom{a}{k} x^k + x^{n+1}A(x) \qquad \textbf{(E7)}$$

$a = \frac{1}{2}$: $\quad \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + x^3 A(x)$

$a = -\frac{1}{2}$: $\quad \frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + x^3 A(x)$

$a = -1$ $\quad \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^5 + \ldots + (-1)^n x^n + (-1)^{n+1} \frac{x^{n+1}}{1+x} \qquad x \neq -1$

OBS: $A(x)$ är en begränsad funktion i en omgivning av 0.