(1) Compute the Riemannian volume of S^2 with the round metric and T^2 with the flat metric.
(2) Using the Lie derivative, find the Killing vector fields of the hyperbolic half plane. Then compute their flows.
(3) Show that vertical lines as geodesics of the hyperbolic half plane, are defined on all of $\mathbb{R}$.
(4) Show that for a differential form ω on a manifold M and a vector field X on M, we have
$$L_X\omega = d\iota_X\omega + \iota_X d\omega.$$
(5) Show that geodesics of a compact *linear* Lie group with the bi-invariant metric are one parameter subgroups, i.e. of the form $\gamma(t) = g\exp(tX)$ for $g \in G$ and $X \in T_eG$. (So the Lie-group-theoretic exponential map agrees with the Riemannian exponential for the bi-invariant metric.)
(6) If ∇ denotes the Levi-Civita connection for the bi-invariant metric on a compact Lie group G and X, Y, Z are left invariant vector fields on G then show that
$$\nabla_X Y = \frac{1}{2}[X, Y].$$
Compute the Riemannian curvature $R(X, Y)Z$.

(7) Show that in the normal coordinates $(U, x_1, \ldots, x_n)$ around a point $p \in (M, g)$ the Riemannian curvature at p is given by
$$R_{ijkl} = -\frac{1}{2}(\partial_i\partial_k g_{jl} - \partial_i\partial_l g_{jk} - \partial_j\partial_k g_{il} + \partial_j\partial_l g_{ik}).$$
This is Riemann's original definition of curvature.