(1) Show that the two charts on S^2 given by streographic projection, constitute a smooth atlas.
(2) Which one of the complex line bundles L_n on S^2 is (isomorphic to) TS^2? Which one to T^*S^2? Prove.
(3) Prove that for α, β differential forms on N and $\phi : M \to N$ a smooth map, $\phi^*(\alpha \wedge \beta) = \phi^*\alpha \wedge \phi^*\beta$.
(4) The hyperboloid model for the hyperbolic n-space is given as follows. Consider the (non-definite) inner product on $\mathbb{R}^{n+1}$ given by

$$< x, y >= -x_0 y_0 + \sum_{i \geq 1} x_i y_i$$

(called the Minkowski metric) and let $H^n = \{x \in \mathbb{R}^{n+1} : x_0 > 0, < x, x >= -1\}$. Show that the restriction of $<, >$ to H^n gives a Riemannian metric g on H^n.
(5) The disk model of the hyperbolic space is given by $B^n = \{y \in \mathbb{R}^n : ||y|| \leq 1\}$ and the metric

$$h = \frac{4}{(1 - ||y||^2)^2}(dy^1 \otimes dy^1 + \cdots + dy^n \otimes dy^n).$$

Show that the map $\phi : H^n \to B^n$ given by

$$\phi(x) = \frac{1}{1 + x_0}(x_1, \ldots, x_n)$$

is an isometry between (H^n, g) and (B^n, h).
(6) Show that an isometry of the Euclidean space which fixes the origin, has to be a linear map.