BASKURS
TENTA
2010-10-08

(1).

$$1<\frac{2x-7}{x+1}\le 3 \Leftrightarrow 1<\frac{2x-7}{x+1} \text{ och } \frac{2x-7}{x+1}\le 3$$

a) $1<\frac{2x-7}{x+1} \Leftrightarrow \frac{2x-7}{x+1}-1>0 \Leftrightarrow \frac{2x-7-(x+1)}{x+1}>0 \Leftrightarrow$

$\Leftrightarrow \frac{x-8}{x+1}>0$. Teckenschema:

x		-1		8	
$x-8$	$-$	$-$	$-$	0	$+$
$x+1$	$-$	0	$+$	$+$	$+$
$\frac{x-8}{x+1}$	$+$	⧘	$-$	0	$+$

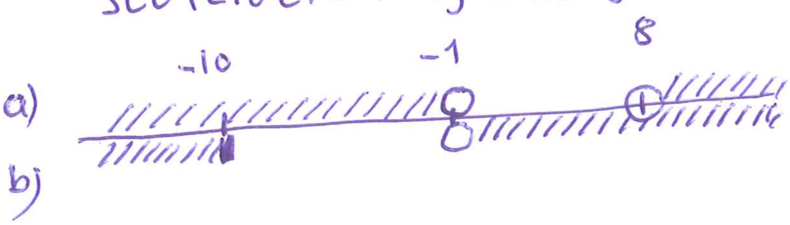
b) $\frac{2x-7}{x+1}\le 3 \Leftrightarrow \frac{2x-7}{x+1}-3\le 0 \Leftrightarrow$

$\frac{2x-7-3(x+1)}{x+1}\le 0 \Leftrightarrow \frac{-x-10}{x+1}\le 0$

$\Leftrightarrow \frac{-(x+10)}{x+1}\le 0$ Teckenschema:

x		-10		-1	
$-(x+10)$	$+$	0	$-$	$-$	$-$
$x+1$	$-$	$-$	$-$	0	$+$
$\frac{-(x+10)}{x+1}$	$-$	0	$+$	⧘	$-$

SLUTLIGEN: a) och b):



Svar: Olikheten gäller för $x\le -10$, $x>8$

(2) $(e^{3x}-e)(\ln(3x+6)-2\ln(x+2))=0 \Leftrightarrow$

$(e^{3x}-e)=0$ eller $\ln(3x+6)-2\ln(x+2)=0$

a) $e^{3x}-e=0 \Leftrightarrow e^{3x}=e \Leftrightarrow e^{3x-1}=1 \Leftrightarrow 3x-1=0$

$\Leftrightarrow \boxed{x=1/3}$

b) $\ln(3x+6) - 2\ln(x+2) = 0 \Leftrightarrow$

$\ln(3x+6) - \ln(x+2)^2 = 0 \Leftrightarrow$

$\ln \dfrac{3x+6}{(x+2)^2} = 0 \Leftrightarrow \ln \dfrac{3(x+2)}{(x+2)^2} = 0 \Leftrightarrow$ (Förkorta!)

$\Leftrightarrow \ln \dfrac{3}{x+2} = 0 \Leftrightarrow \dfrac{3}{x+2} = 1 \Leftrightarrow$

$\Leftrightarrow 3 = x+2 \Leftrightarrow \boxed{x=1}$.

Svar: $\boxed{x = 1/3 \,,\ x = 1}$

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③ $\displaystyle\sum_{k=0}^{n} \frac{2-k}{2^k} = 2 + \frac{n}{2^n}$, $n \in \mathbb{N}$.

Basfall: $n=0$. $VL = \displaystyle\sum_{k=0}^{0} \frac{2-k}{2^k} = \frac{2-0}{2^0} = 2$; $HL = 2 - \dfrac{0}{2^0} = 2$. $VL = HL$.

Induktionssteg:

Vi vill visa att om för ett visst n_0 gäller

(A) $\displaystyle\sum_{k=0}^{n_0} \frac{2-k}{2^k} = 2 + \frac{n_0}{2^{n_0}}$ så följer $\displaystyle\sum_{k=0}^{n_0+1} \frac{2-k}{2^k} = 2 + \frac{(n_0+1)}{2^{n_0+1}}$ (B)

$VL(B) = \displaystyle\sum_{k=0}^{n_0} \frac{2-k}{2^k} + \frac{2-(n_0+1)}{2^{n_0+1}} =$ (Eftersom (A) antas gälla) $=$

$= \left(2 + \dfrac{n_0}{2^{n_0}}\right) + \dfrac{2-n_0-1}{2^{n_0+1}} = 2 + \dfrac{2n_0 + 2 - n_0 - 1}{2^{n_0+1}} = 2 + \dfrac{n_0+1}{2^{n_0+1}} =$

$= HL(B)$, så induktionssteget gäller.

Induktionsaxiomet ger slutligen att påståendet är sant för alla $n \in \mathbb{N}$. $\square$

4) a) $$\sum_{k=1}^{7}(-7+(k-1)4)$$

b) $$\sum_{p=0}^{10} 2\left(\frac{1}{3}\right)^p = 2\sum_{p=0}^{10}\left(\frac{1}{3}\right)^p = 2\cdot\frac{1-\left(\frac{1}{3}\right)^{11}}{1-\frac{1}{3}} =$$

$$= 2\cdot\frac{1-\frac{1}{3^{11}}}{\frac{2}{3}} = \boxed{3-\frac{1}{3^{10}}}$$

5) Vi skriver alla talen på polär form:

- $-4i = 4(0+(-1)\cdot i) = \underline{4\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)}$.
- $6+i\sqrt{12} = \left(|6+i\sqrt{12}| = \sqrt{36+12} = \sqrt{48} = 4\sqrt{3}\right) =$
  $= 4\sqrt{3}\left(\frac{6}{4\sqrt{3}}+i\frac{\sqrt{12}}{4\sqrt{3}}\right) = 4\sqrt{3}\left(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right) = \underline{4\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}$.
- $\sqrt{3}-3i = \left(|\sqrt{3}-3i| = \sqrt{3+3^2} = \sqrt{12} = 2\sqrt{3}\right) = 2\sqrt{3}\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) =$
  $= \underline{2\sqrt{3}\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)}$.
- $1+i = \left(|1+i| = \sqrt{1+1} = \sqrt{2}\right) = \sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) = \underline{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}$

Beloppet blir: $\frac{4\cdot 4\sqrt{3}}{2\sqrt{3}\cdot\sqrt{2}} = \boxed{\frac{8}{\sqrt{3}}}$

Argumentet: $\left(-\frac{\pi}{2}\right)+\frac{\pi}{6}-\left(-\frac{\pi}{3}\right)-\frac{\pi}{4} = -\frac{\pi}{4}$

Talet blir: $\boxed{\frac{8}{\sqrt{3}}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)}$

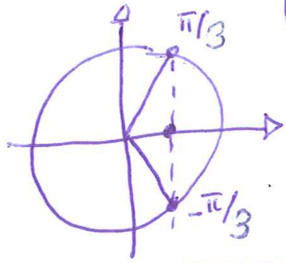
⑥ $\cos 2x + 3\cos x = 1 \Leftrightarrow (2\cos^2 x - 1) + 3\cos x = 1 \Leftrightarrow$

[DUBBLA VINKELN]

$2\cos^2 x + 3\cos x - 2 = 0 \Leftrightarrow \{\cos x = t\} \Leftrightarrow 2t^2 + 3t - 2 = 0$ OMÖJLIGT!

$\Leftrightarrow t^2 + \frac{3}{2}t - 1 = 0 \Leftrightarrow t = -\frac{3}{4} \pm \sqrt{\frac{9}{16} + 1} = -\frac{3}{4} \pm \frac{5}{4} = \begin{cases} -2 \\ 1/2 \end{cases}$

Så $\cos x = 1/2 \Rightarrow \boxed{x = \pm \frac{\pi}{3} + 2n\pi}$



⑦ $\left(x^2 - \frac{3}{x}\right)^{17} = \sum_{k=0}^{17} \binom{17}{k} (x^2)^k \left(\frac{3}{x}\right)^{17-k} =$

$= \sum_{k=0}^{17} \binom{17}{k} \frac{x^{2k} \cdot 3^{17-k}}{x^{17-k}} =$

$= \sum_{k=0}^{17} \binom{17}{k} \cdot x^{3k-17} \cdot 3^{17-k}$

För termen som innehåller x^{31} är $3k - 17 = 31 \Leftrightarrow 3k = 48$ $\Leftrightarrow \boxed{k = 16}$

$k=16$ ger $\binom{17}{16} \cdot 3^{17-16} = \frac{17!}{16! \cdot 1!} \cdot 3 = 3 \cdot 17 = \boxed{51}$ SVAR!!

⑧ Sätt in $z = ai$: $2(ai)^4 + 2(ai)^3 - 11(ai)^2 + ai = 6 \Leftrightarrow$

$\begin{cases} 2a^4 + 11a^2 - 6 = 0 \quad (1) \\ -2a^3 + a = 0 \Leftrightarrow -a(2a^2 - 1) = 0, \ \boxed{a = \pm 1/\sqrt{2}} \end{cases}$ ($a=0$ löser ej (1)!)

SLUTSATS: $z = \pm \frac{i}{\sqrt{2}}$ är rötter, och enligt faktorsatsen är polynomet delbart med $(z - \frac{1}{\sqrt{2}}i)(z + \frac{1}{\sqrt{2}}i) = (z^2 + \frac{1}{2})$ eller $\boxed{2z^2 + 1}$. Dividera:

$2z^4 + 2z^3 - 11z^2 + z - 6 = (2z^2 + 1)(z^2 + z - 6)$

SLUTLIGEN: $z^2 + z - 6 = 0 \Leftrightarrow z = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} = -\frac{1}{2} \pm \frac{5}{2} = \begin{cases} -3 \\ 2 \end{cases}$

Svar: Rötterna är $\boxed{\pm \frac{i}{\sqrt{2}}, 2, -3.}$