On observational effects of a varying evolution parameter in standard cosmology and the Hoyle-Narlikar cosmology

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Abstract

The effects of a time-dependent evolution parameter on the m versus z relation and the $N(m)$ versus m relations are derived, for standard cosmology and for the Hoyle-Narlikar theory.

1. Introduction

Canuto and Narlikar (1980) derive various observationally relevant relations for the Hoyle-Narlikar cosmology. They attempt to eliminate the evolution parameter between the m versus z relation and the $N(m)$ versus m relation and obtain a relation involving only observable quantities. However, their derivation is incorrect since the relation obtained for QSO:s uses a variable evolution parameter e, while e is assumed to be constant in the derivation.

The purpose of this paper is to find the correct form of the m versus z and $N(m)$ versus m relations for the case of a time-dependent e. This is of interest since comparisons with observations is the only way of testing the validity of cosmologies. The basic calculations will be carried out for an arbitrary Robertson-Walker metric. The results will later be specialized to the standard cosmology and the Hoyle-Narlikar theory. A numerical computation gives results differing from the predictions by Canuto and Narlikar (1980) by 0.7 magnitudes for distant QSO:s.

2. Basic theory

We assume that the objects under study have absolute luminosities given by

$$L(t) = L_0(t/t_0)^{-e(t)} . \qquad (2.1)$$

Note that any luminosity function may be written as in (2.1) with some function $e(t)$. We prefer to work with $e(t)$ rather than $L(t)$ since this faciliates comparisons with the case of a constant e . (Presumably, $e(t)$ varies slowly.)

Note also that, for a given luminosity function $L(t)$, $e(t)$ depends on the present epoch t_0 . The evolution during a small interval of time $(t, t+dt)$ is not given by $e(t)$ but by the logarithmic derivative

$$e_1(t) = -\frac{d\ \ell n\ L(t)}{d\ \ell n(t)} = e(t) + \dot{e}(t)t\ \ell n(t/t_0) . \qquad (2.2)$$

(In this paper ℓn denotes natural logarithms and $\log = {}^{10}\log$.)
$e_1(t)$ does not depend on t_0 . Given $e_1(t)$, $L(t)$ and $e(t)$ may be obtained by integration. Obviously, $e_1 = e$ when e is constant. (It is the failure to distinguish between e and e_1 that makes the analysis in Canuto and Narlikar 1980 for QSO:s invalid.)

We assume a standard Robertson-Walker metric

$$d\tau^2 = dt^2 - \frac{R^2(t)}{c^2}\left(\frac{d\sigma^2}{1-k\sigma^2} + \sigma^2 d\theta^2 + \sigma^2 \sin^2\theta\ d\varphi^2\right) \qquad (2.3)$$

$$(\sigma_{earth} = 0)$$

and will derive relations between the time t when the light now observed was emitted, the coordinate σ , the apparent magnitude m and other properties of objects with the absolute luminosity (2.1). The redshift is given by the well-known formula

$$1 + z = R_0/R(t) \, . \tag{2.4}$$

In the standard Friedmann models, the apparent luminosity is given by

$$\ell = \frac{L(t)}{4\pi (R_0\sigma)^2 (1+z)^2} = \frac{L(t)\ R^2(t)}{4\pi\ R_0^4\ \sigma^2} \, . \tag{2.5}$$

In the Hoyle-Narlikar theory, photons are not conserved and a further factor $G(t)/G_0 = t_0/t$ enters, cf. Canuto and Narlikar (1980). Following these authors we write $G(t)/G_0 = (t/t_0)^{-g}$, where $g = 0$ for general relativity and $g = 1$ for the Hoyle-Narlikar theory. Thus

$$\ell = \frac{L_0}{4\pi\ R_0^4} \frac{R^2(t)}{\sigma^2} \left(\frac{t}{t_0}\right)^{-e(t)-g} \tag{2.6}$$

i.e.

$$m = m_0 + 5 \log\sigma - 5 \log R + 2.5(e+g)\log(t/t_0) \, . \tag{2.7}$$

Since light travels along a null geodesic

$$\frac{d\sigma}{dt} = - \frac{c\sqrt{1-k\sigma^2}}{R(t)} \tag{2.8}$$

(2.7), (2.8) and (2.2) yield

$$\ell n\ 10 \frac{dm}{dt} = -5 \frac{c\sqrt{1-k\sigma^2}}{\sigma R(t)} - 5 \frac{\dot{R}}{R} + 2.5 \frac{e_1 + g}{t} \, . \tag{2.9}$$

We assume that dm/dt is negative so that more distant objects are dimmer. The total number of objects with magnitude not exceeding m then is given by the (present-day) density n_0 multiplied by the volume of the sphere with coordinate radius σ:

$$N(m) = n_0 \int_0^\sigma 4\pi\ R_0^3\ s^2 \frac{ds}{\sqrt{1-ks^2}} \, . \tag{2.10}$$

Consequently,

$$\frac{dN}{d\sigma} = 4\pi\ n_0\ R_0^3 \sigma^2 \,/\, \sqrt{1-k\sigma^2} \, . \tag{2.11}$$

The chain rule and (2.8) - (2.11) yield

$$\frac{d\ \log N}{dm} = \frac{1}{\ln 10}\,\frac{1}{N}\,\frac{dN}{d\sigma}\,\frac{d\sigma}{dt}\Big/\frac{dm}{dt} =$$

$$= \frac{\sigma^2}{\int_0^\sigma s^2(1-ks^2)^{-1/2}ds}\,\frac{c}{R(t)}\left(5\,\frac{c\sqrt{1-k\sigma^2}}{\sigma R(t)} + 5\,\frac{\dot{R}}{R} - 2.5\,\frac{e_1+g}{t}\right)^{-1} =$$

$$= \frac{3}{5}\,\frac{\sigma^3}{3\int_0^\sigma s^2(1-ks^2)^{-1/2}ds}\left(\sqrt{1-k\sigma^2} + \frac{\sigma\dot{R}}{c} - \frac{1}{2}\,\frac{\sigma R}{ct}\,(e_1+g)\right)^{-1}. \qquad (2.12)$$

When the space is flat, $k=0$, this simplifies to

$$\frac{d\log N(m)}{dm} = \frac{3}{5}\,\left(1 + \frac{\sigma\dot{R}}{c} - \frac{1}{2}\,\frac{\sigma R}{ct}\,(e_1+g)\right)^{-1}. \qquad (2.13)$$

3. Standard cosmology

In the Friedmann model with $k=0$, $R/R_0 = (t/t_0)^{2/3}$ whence $\sigma R_0 = 3ct_0(1-(1+z)^{-1/2})$, $\sigma\dot{R}/c = 2(\sqrt{1+z}-1)$ and $\sigma R/ct = 3(\sqrt{1+z}-1)$. Thus (since $g=0$) we obtain from (2.13)

$$\frac{d\log N(m)}{dm} = \frac{0.6}{1+(2-1.5\,e_1)(\sqrt{1+z}-1)} \,. \tag{3.1}$$

m is expressed in z and e or e_1 by

$$m = m_0' + 5\log(1+z-(1+z)^{1/2}) - 3.75\,e\log(1+z) =$$

$$= m_0' + 5\log(1+z-(1+z)^{1/2}) - \frac{3.75}{\ln 10}\int_0^z \frac{e_1(z)}{1+z}\,dz \,. \tag{3.2}$$

For the models with $k=\pm 1$ no explicit formulas are available, but numerical results are readily obtained.

4. Hoyle-Narlikar cosmology

In the Hoyle-Narlikar theory $k=0$ and $R = R_0(t/t_0)^{1/2}$ whence $\sigma R_0 = 2ct_0 z/(1+z)$ (Canuto and Narlikar 1980). Thus $\frac{\sigma\dot{R}}{c} = z$ and $\frac{\sigma R}{ct} = 2z$. (2.13) yields (since $g=1$)

$$\frac{d \log N(m)}{dm} = \frac{0.6}{1+(1-e_1-g)z} = \frac{0.6}{1-e_1 z} . \tag{4.1}$$

(This is the formula derived by Canuto and Narlikar (1980) but with e_1 replacing e .)

(2.7) may be rewritten as

$$m = m_0' + 5 \log cz + 2.5(e+g)\log(t/t_0) =$$

$$= m_0' + 5 \log cz - 5g \log(1+z) - \frac{2.5}{\ln 10} \int_t^{t_0} \frac{e_1(t)}{t} dt =$$

$$= m_0' + 5 \log cz - 5 \log(1+z) - \frac{5}{\ln 10} \int_0^z \frac{e_1(z)}{1+z} dz . \tag{4.2}$$

Canuto and Narlikar (1980) take $\frac{d \log N(m)}{dm} = 0.75$ for QSO:s and derive $e = \frac{1}{5z}$ (for $z \geq 1/3$) . We obtain from (4.1) instead

$$e_1 = \frac{1}{5z} \quad \text{(for QSO:s, } z \geq 1/3\text{)}. \tag{4.3}$$

(4.2) and (4.3) yield, for the relevant range of z ,

$$m = m_0'' + 4\log(cz) - 4\log(1+z) . \tag{4.4}$$

If we adjust the constant term such that this coincides with Canuto and Narlikar's formula (with $e=1/5z$) for $z = 1/3$, we obtain significantly smaller magnitudes. Numerical values of the difference are given below.

z	$m_{(4.4)} - m_{\text{Canuto and Narlikar}}$
0.5	-0.1
1.0	-0.4
1.5	-0.5
2.0	-0.6
3.0	-0.7

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Reference

Canuto, V.M. and Narlikar, J.V. 1980, Ap.J. 236, 6-23.