THE ICELANDIC CALENDAR

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Abstract. The Icelandic calendar, which for centuries was the civil calendar on Iceland, has a year of 52 weeks, that is 364 days; this is kept in step with the tropical year, and thus with the seasons, by the intercalation of a leap week some years. The basic subunit is the week; dates were traditionally given by the day of week and a counting of the number of weeks. There is also a division of the year into 12 months of 30 days each plus 4 extra days.

1. Introduction

Iceland has a unique calendar, which was used as the civil calendar by the general population from the 10th to the 18th century, and occasionally later; it is still included in the Icelandic Almanac [2]. The purpose of this paper is to give a detailed description (in English) of this calendar and its historical development, including some mathematical formulas for calculating the calendar.

The Icelandic calendar has ordinarily a year of 364 days, that is exactly 52 weeks; some years are leap years with a *leap week* (Icel. *sumarauki* = 'summer increase'), making the leap year 371 days = 53 weeks.[1]

Every year is thus a whole number of weeks, and consequently every year begins on the same day of week. The year is divided into 12 months, listed in Table 1[2]. Each month has 30 days, and there are 4 extra days (*aukanœtur*), or in leap years 11 extra days (*aukanœtur* + *sumarauki*), between the third and fourth summer months. Hence each month begins on the same day of week (given in Table 1) every year. (See Section 7.1 for a different placement of the leap week for some period until 1928, and Section 7.2 for an alternative, but probably incorrect, description in some references.)

The leap weeks are, since the 12th century, inserted when necessary so that the beginnings of the months fall in the ranges given in Table 1 in the Julian (before 1700) or Gregorian (after 1700) calendar, see further Section 3

Date: 24 November 2009; Tuesday, 5th week of winter; 2nd day of Ýlir; revised 8 December 2010.

Key words and phrases. calendar; weeks; months.

[1] In this paper we use the English term "leap year" for the Icelandic calendar, as well as "leap week". Note, however, that in the Icelandic literature, "leap year" (*hlaupár*) always means a leap year in the Julian or Gregorian calendar.

[2] In this paper we use old forms such as Góa when discussing medieval times and modern forms such as Góa for modern times.

	old Icelandic	modern Icelandic	begins	Julian	Gregorian
S1	?	Harpa	Thursday	9–15 April	19–25 April
S2	?	Skerpla	Saturday	9–15 May	19–25 May
S3	?	Sólmánuður	Monday	8–14 June	18–24 June
S4	(Miðsumar)?	Heyannir	Sunday	13–20 July	23–30 July
S5	Tvímánuðr	Tvímánuður	Tuesday	12–19 August	22–29 August
S6	?	Haustmánuður	Thursday	11–18 September	21–28 September
W1	Gormánuðr	Gormánuður	Saturday	11–18 October	21–28 October
W2	Ýlir	Ýlir	Monday	10–17 November	20–27 November
W3	Jólmánuðr	Mörsugur	Wednesday	10–17 December	20–27 December
W4	Þorri	Þorri	Friday	9–16 January	19–26 January
W5	Góa	Góa	Sunday	8–15 February	18–25 February
W6	Einmánuðr	Einmánuður	Tuesday	10–16 March	20–26 March

TABLE 1. The Icelandic months. See also Table 2 for alternative names.

for the history and Sections 4–6 for calculations. (Note that the ranges are 7 days for some months and 8 days for others, see Section 4.1.) Further formulas and comments are given in Appendices B–E.

The year is divided into two halves (semesters or seasons, Icel. *misseri*): summer (*sumar*) and winter (*vetur*). This was originally a fundamental division of the year, marked by the *First Day of Summer* (*Sumardagurinn fyrsti*, always a Thursday) and the *First Day of Winter* (*Fyrsti vetrardagur*, now always a Saturday, but earlier sometimes a Friday, see Section 2.1); months and weeks were counted from the beginning of the *misseri*, or backwards from the end of them. In the present paper, the months in Table 1 are therefore numbered as S1–S6 (summer months) and W1–W6 (winter months).

Note that the summer is 184 or 191 days and the winter is 180. Thus the summer is slightly longer, and none of the *misseri* is a whole number of weeks.[3]

There is no special numbering of the Icelandic years.

Acknowledgement. I thank Elísabet Andrésdóttir, Þorsteinn Sæmundsson, Þorsteinn Vilhjálmsson, Henrik Williams and two anonymous referees for comments and other help.

[3]As noted by Beckman [5, Tab. VIII], the fact that the summer *misseri* is longer than the winter is in accordance with the astronomical fact that the summer half-year between the equinoxes is longer than the winter half-year (about 186 vs 179 days). However, there is no evidence that this was known in Iceland in the 12th century (or earlier). On the contrary, a 12th century text attributed to Stjǫrnu-Oddi ("Star-Oddi") Helgason seems to show that he assumed that the solstices and equinoxes are equally spaced [35]. Beckman [5, Tab. VIII] further notes that in the Middle Ages, the equinoxes were close to the beginnings of W6 and S6. Again, this seems to be a coincidence, since as far as I know, there are no old Icelandic texts or comments mentioning (or hinting at) a connection.

	Snorri's *Edda* [32]	*Bókarbót* [1, p. 78]	modern names
S1	Gaukmánuðr, Sáðtið		Harpa
S2	Eggtið, Stekktið		Skerpla
S3	Sólmánuðr, Selmánuðr		Sólmánuður
S4	Heyannir		Heyannir
S5	Kornskurðarmánuðr	Tvímánuðr	Tvímánuður
S6	Haustmánuðr		Haustmánuður
W1	Gormánuðr	Gormánuðr	Gormánuður
W2	Frermánuðr	Ýlir	Ýlir
W3	Hrútmánuðr	Mǫrsugr	Mörsugur
W4	Þorri	Þorri	Þorri
W5	Góa	Góa	Góa
W6	Einmánuðr	Einmánuðr	Einmánuður

TABLE 2. Further old Icelandic month names, many of them probably never used in practice.

2. Subdivisions of the year

2.1. ***Misseri* (semesters).** As said above, the year is divided into two *misseri*: summer and winter.[4] The importance of the *misseri* is shown by the fact that the Icelandic calendar is called *misseristal* in Icelandic. Some further examples of the importance of the *misseri* is that *Rím I* [1, pp. 3–64] (written in the late 12th century [5, p. 13–14]) begins a description of the Icelandic year with "This is *misseri* reckoning, that 2 *misseri* are called a year, that is winter and summer."[5], and this is repeated in the later *Rím II* [1, pp. 83–178] (13th century [1, p. XCVIII]) as "Two *misseri* are called a year, that is winter and summer."[6]; furthermore, *Íslendingabók* [3, 19] (a brief Icelandic history written by Ari hinn fróði ("Ari the Wise") c. 1125) begins the story of the leap weeks with "in two *misseri* 364 days"[7] (although "year" also is used later in the story). Also, the standard way of expressing ages of persons or other periods of years was using "winters" [17, p. 58 §229]. (Sometimes, "summers" was used instead. For example, [3] has several examples like "130 winters" and "20 summers".)

I have not found any clear evidence of whether the summer or winter *misseri* comes first in a year. (I do not even know if the question would have made sense to a medieval Icelander, since the years were not numbered. No day was celebrated as New Year [9, pp. 14, 83].) Usage may have been

[4]Spring and autumn existed as seasons, but they did not have any function in the calendar. They were not precisely defined, although Snorri attempts exact definitions in *Edda* [32, Skáldskaparmál 79, p. 179], see [5, p. 35] and [31, p. 307].

[5]Þath er missaris tal, ath II missere heiter ær, þat er vetr ok sumar. [1, p. 22 §26]

[6]Tváv misseri heita ár, þat er vetr ok svmar. [1, p. 83 §2]

[7]itveim misserom fiora daga ens fiorþa hundraþs [3, Ch. IV]

varying, and both winter+summer and summer+winter may have been considered as a year.[8] In the quotes above from *Rím I* and *Rím II* [1], winter is mentioned before summer. On the other hand, the following discussions in *Rím I* and *Rím II* of the year and its various holidays and other important dates start with the beginning of summer and continue until the end of winter. For this reason, I have (somewhat arbitrarily) chosen to define the year as starting with the summer half in this paper.[9]

Friday or Saturday? While everyone agrees that summer begins on a Thursday, there are two different traditions for the beginning of winter: Friday or Saturday. The learned medieval literature (for example the computistic texts in [1] and the laws *Grágás* [18]) clearly specifies Saturday, see for example *Bókarbót*: "Winter and Gormánuðr come on a Saturday."[10] and the quote from *Grágás* in Section 2.3 (footnote 18) This also follows if there are 6 months with $6 \cdot 30 + 4$ (or $6 \cdot 30 + 11$) days in the summer, and if there are 6 months of 30 days each in the winter. However, the winter was reckoned from a Friday (one day before the beginning of winter as shown in Table 1) from the 16th century until the Icelandic Almanac began to be published in 1837 when the Saturday reckoning was revived, see [10], [31, p. 320 and p. 330], [5, p. 35]. (In particular, see [31, p. 320 n. 3] for an account by Briem on popular opposition to the change to Saturday.) For example, the law made in 1700 [23, p. 1376] concerning the change to the Gregorian calendar explicitly reckons winter from a Friday, see Section 3.4.

The Friday beginning is first documented in 1508, and 16th century use seems mixed [39]: A later 16th century document says that winter begins on a Saturday except at *rímspillir* (see Section 4.1) when it begins on a Friday (this rule keeps the beginning of winter to the week 11–17 October, see Tables 1 and 5). Another document, from 1589, says that the farmers reckoned the winter from a Friday, but that the correct reckoning according to books is from a Saturday.

Björnsson [10] and Beckman [1] believe that Friday was the original day; Beckman [1, p. LXXII] suggests that it was moved to Saturday as part of a 12th century calendar reform (to conform with 30 days months), but that it took some 700 years for this change to gain acceptance. However,

[8]Sæmundsson [30] mentions a third possibility that has been discussed, namely that the year began at Midsummer; *Rím II* discusses the months in this order at one place [1, pp. 138–139 §113], and this would put the extra days and the leap week at a natural place at the end of the year.

[9]Today, when the First Day of Summer is celebrated as a public holiday in Iceland, but the First Day of Winter is not, it also seems natural to start with the summer. Björnsson [9] also regards the First Day of Summer as the beginning of the year. On the other hand, Schroeter [31, p. 308] finds it probable that summer came last, and Ginzel [17, p. 58] claims that the year began with winter. Moreover, the law from 1700 discussed in Section 3.4 discusses primarily the First Day of Winter; I do not know whether this indicates that it was at that time regarded as more fundamental than the First Day of Summer.

[10]Vetr oc gormanuðr keмr laugar dag. [1, p. 78]

Þorkelsson [39, pp. 59–63] finds no evidence of this and argues that the Friday reckoning was introduced around 1500; see also [22, Første vinterdag, sommerdag] where it is suggested that the change to Friday was due to a mistake.

2.2. **Weeks.** The standard way of reckoning time was using weeks. (The dominance of the week is reflected in the construction of the calendar with a whole number of weeks every year.)

Weeks were used both to measure time intervals, and to specify dates by giving the week and day of week. There are many examples of this in both the literature (for example, the Icelandic Sagas) and the medieval laws *Grágás* [18] (written down c. 1250), as well as later. Some medieval examples can be seen in Appendix D; further examples (from *Grágás*) are given by Björnsson [10, pp. 277–279] (although some of his conclusions seem to be unfounded, see Beckman [1, p. LXXIII–LXXIV]). See also Schroeter [31, pp. 327–331].

There is no single fixed day of week beginning the weeks. One method is to number the weeks in each *misseri* from the beginning of the *misseri*, with the first week starting on the first day of the *misseri*. Thus summer weeks begin on Thursdays, and are numbered from 1 to 26 (or 27 in leap years, and ignoring the last two days, which are called *veturnætur*), and winter weeks begin on Saturdays (or Fridays, see Section 2.1) and are numbered from 1 to 26, with the last week incomplete. This is the modern method, given in the Icelandic Almanac [2].

In earlier days, this method was used for the first half of each *misseri*. In the second half, that is after Midsummer or Midwinter, weeks were counted backwards from the end of the *misseri*, or the number of remaining weeks was given. (This keeps the numbers small, at most 13 or possibly 14.) An example is given in the rule on Ember days in Appendix D. According to Schroeter [31, p. 328], this was in the early days a firm rule. However, there were also other possibilities. Weeks were sometimes counted from Midsummer or Midwinter, or from some other day. Þorkelsson [39, p. 52] says that weeks in the second half of the winter, except the three last, were numbered using the months Þorri and Góa. (An example can be seen in the rule on Candlemas in Appendix D.)[11] [12]

[11] I do not know from which day of week these weeks were reckoned; a possibility is that weeks could start on different days and that the rule stated above (and in for example [11, p. 23] and [28]) that summer weeks begin on Thursdays and winter weeks begin on Saturdays is valid only for weeks counted from the beginning of the *misseri*, and that conversely winter weeks counted backwards from the First Day of Summer begin on a Thursday and summer weeks counted backwards from the First Day of Winter begin on a Saturday. But my research has been incomplete and I have not found any specific reference or evidence for this.

[12] Schroeter [31, p. 331] also claims (with a reference to *Vigfússon: Corpus Poeticum Boreale*) a version, starting at Christmas, reckoning with the first and second weeks of Þorri beginning of Fridays (that is 1–7 Þorri and 8–14 Þorri) but the third and fourth

There are also examples of reckoning weeks from Sundays, following the Christian church (and in accordance with the names of the days of week, see below); see for example the rules on Christmas, Candlemas and First Day of Summer in Appendix D. In fact, *Rím II* says: "Sunday is first in the week in day reckoning and in *misseri* reckoning [the Icelandic calendar], but various days in month reckoning."[13] However, this is, as seen above, usually not the rule for weeks reckoned with the Icelandic calendar.

The Icelandic names of the days of week (used for both the Icelandic calendar and for the Julian and Gregorian) are given in Table 3.[14] Note that in modern Icelandic, Tuesday and Thursday are simply numbered ("Third day" and "Fifth day", counting in the biblical way with Sunday the first day of week); the original names after Germanic gods that are used for Tuesday–Friday in other Scandinavian languages, as well as in English, were opposed on religious grounds by bishop Jón Ögmundsson of Hólar (1052–1121; bishop from 1106)[15], who succeeded to replace them by neutral or Christian (Icel. *föstudagur* = "Fast-day") names.[16] He also introduced *drottinsdagur* ("The Lord's day") for Sunday and *annardagur* ("Second day") for Monday; but these later disappeared again and the old names returned to favour.[17]

2.3. **Months.** The months were originally of very little importance; as said above, the standard way of reckoning time was using weeks.

The months are clearly defined (without names) in *Grágás* [18]: "The first day of summer is to be a Thursday; from then three months of 30 nights and 4 nights in addition are to be counted to Midsummer. From Midsummer there are to be 3 months of 30 nights to winter. The first day of winter is to be a Saturday and from then there shall be 6 months of 30 nights to

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weeks of Þorri beginning on Saturdays, leaving *miðþorri* (15 Þorri) and *þorraþrœll* (30 Þorri) outside the week reckoning; then 4 weeks of Góa are reckoned similarly; then weeks in Einmánuður, without exceptions for the middle and last days; then summer weeks numbered 1–26 (or 27); then 9 winter weeks until Christmas. I cannot say whether this complicated method really has been used in reality. If so, I suspect that it should be regarded as a combination showing that different methods coexisted (possibly with different versions dominating in different parts of the year), rather than a fixed rule.

[13]Drottins dagr er fystur i viku at daga tali ok misseris tali, en ymser dagar at manadar tali. [1, p. 128 §83]

[14]Old names after [31, pp. 307, 319–320]; see also [1, Register I].

[15]Bishop Jón was declared a saint by the Althingi in 1200 [9]. Perhaps this was mainly for other deeds.

[16]*Rím I* and *Rím II* use almost exclusively the new names. Exceptions are [1, pp. 84 §6] (*frjádagr*) and the final paragraphs of *Rím II* [1, pp. 175–178 §§181–182] (where the old names are used in predictions of the next year's weather based on the day of week of Christmas Day). *Rím I* [1, p. 63 §79] also gives the connection between days of week (using the new names) and the Germanic and Latin gods and planets.

[17]*Drottinsdagur* and *annardagur* are used in, for example, *Rím I*, *Bókarbót* and (*drottinsdagur* only) *Rím II*, as can be seen in several of the quotes in this paper, for example in footnotes 13, 57 and 67.

– c. 1200	modern Icelandic	English
sunnudagr	sunnudagur	Sunday
mánadagr	mánudagur	Monday
týrsdagr	þriðjudagur	Tuesday
óðinsdagr	miðvikudagur	Wednesday
þorsdagr	fimmtudagur	Thursday
frjádagr	föstudagur	Friday
laugardagr	laugardagur	Saturday
or þváttdagr		

TABLE 3. Icelandic names for days of week

summer."[18] Nevertheless, *Grágás* usually reckons in weeks. Similarly, *Rím I* [1, pp. 24–25 §§28–30] mentions Þorri and Góa, but generally uses weeks to describe various important days through the year. Beckman [5, p. 34] knows only a single example in the old Icelandic literature (except texts on time-reckoning) where a date is given using an Icelandic month: "That happened in the last day of Góa, then there were three weeks to Easter."[19] in the Sturlunga Saga (but only in a couple of the existing manuscripts). Otherwise, when months are used at all, they refer to a period of time, and it is often not even clear whether they refer to a calendar month of 30 days or another period, perhaps not precisely defined. The months are mainly mentioned in learned discussions, and it has been suggested that the 30-day months originally were learned constructions without popular use, see Björnsson [10] and Beckman [5]; however, Þorkelsson [39] argues that there would be no reason to construct them unless months already were in use, and views them as relics of older lunar months, see Section 3.1. See further Beckman [5] and Þorkelsson [39].

In fact, the months are used so little in the preserved medieval texts that even the original names of many of them are not known. Sometimes the months are numbered instead, and it is perhaps not clear that all even had names originally. *Bókarbót* [1, p. 78] (12th century) lists for all twelve months the days of week they begin with (in accordance with Table 1), but most of the summer months are numbered and not named. The names that are given in *Bókarbót* are shown in Table 2; they agree with the names in Table 1 except for W3, and also with the modern names. A complete list of month names (sometimes with alternatives) is given by Snorri Sturluson in his *Edda* [32, Skáldskaparmál 79, p. 179] (written c. 1220). These names too are given

[18]En fimti dagr vikv scal vera fyrstr i svmri. þaþan scal telia .iij. manvðr .xxx. nátta. oc nætr .iiij. til mið sumars. En fra miþiv svmri scal .iij. manvþr. xxx. natta til vetrar. Lavgar dagr scal fyrstr vera i vetri. enn þaþan fra scolo vera .vi. manvðr .xxx. natta til sumars. [18, §19 p. 37]

[19]þat var tíðinda enn síðasta dag í góa, þá váru þriár vikur til páscha. [5, p. 34]

in Table 2, but it is doubtful whether these names actually have been used by the general population, see Beckman [5, p. 32].

The old names of the months in Table 1 are given after Beckman [1, 5], who finds them "comparatively well documented", with the addition of Gormánuðr, which also is well documented (for example in *Bókarbót*) although omitted by Beckman for unknown reasons. However, Beckman [1, p. CLXXXVI] and [5] gives Miðsumar for S4, which I have put in parentheses; I am not convinced that Miðsumar really meant a calendar month and not just a period of one or several days in the middle of summer. (It may have been used for both as Beckman [1, Index p. 275] suggests. See also [28, miðsumar].) In the quote from *Grágás* [18, §19] above, it seems to stand for a single day. *Rím I* and *Rím II* [1, pp. 22 §26 and 83–84 §3] say: "From it [First Day of Summer] there should be 13 weeks and 3 days to Midsummer (*Miðsumar*), and a Sunday is the first in Midsummer."[20], which indicates a longer period, although not necessarily a month; however *Rím II* somewhat later says "From Midsummer there are 12 Sundays to winter."[21] which again seems to indicate a single day, and still later "between Sanctorum in Selio [8 July] and Sunday at Midsummer"[22], which could perhaps be interpreted either way. Compare *miður vetur* (Midwinter) which is the first day in the 4th winter month Þorri, but according to Beckman [1, Index p. 275] also could mean the whole month; again I am not convinced. Some examples: *Rím I* says "Midwinter comes three days after the Octave of Epiphany, and is a Friday first in Þorri."[23] (when there is *rímspillir*, see below) which could be interpreted either way, and *Rím II* says "Þorri begins at Midwinter"[24], which seems to mean a single day.

Þorkelsson [39] too believes that the seven names given in Table 1 (excluding Miðsumar) are original; he also finds it possible that Harpa and Skerpla, documented from c. 1600, are old names. See also [22, Månadsnamn].

Only the names of the three last winter months, and in particular Þorri and Góa, seem to have been used frequently. Examples from *Rím I* and *Rím II* with Þorri were just given. *Rím II* names Þorri, Góa and Einmánuðr but no other months in a description of the year [1, p. 139 §116], and the same three, Tvímánuðr and Jólmánuðr in another list of 9 of the months [1, pp. 168–170 §§160–162]. *Grágás* mentions Einmánuðr: "If a chieftain dies before Einmánuðr, ... If he dies after Einmánuðr, ..."[25]. Þorri and Góa appear prominently in the discussions of the beginning of Lent in *Rím I* [1, pp. 25–28 §§30–33] and *Rím II* [1, pp. 139–140 §§116–118], and in the 12th century

[20]Þadan skulo lida XIII vikr ogh III netr til midsumars, ok er drottins dagur fyrstr ath midiu sumre. [1, p. 22 §26]

[21]Fra midiu sumri ero XII drottins dagar til vetrar. [1, p. 84 §5]

[22]i millum Selu mana mesu ok drottins dags at midiu sumri [1, p. 168 §160]

[23]Midr vetur kemr efter enn atta dagh fra hinum þrettanda þrimur nottum, ok er faustu dagur fyrstur i þorra. [1, p. 24 §28]

[24]Þora manudr kemur i midian vetur [1, p. 139 §116]

[25]Nv andaz goði fyrir einmanað ... Nv andaz hann eptir einmanað ... [18, §84 p. 142]

Easter table in [1, pp. 69–70], where the beginning of Lent each year is given by giving the week of Þorri or Góa; see further Appendix C.1.[26]

The popularity of the names Þorri and Góa is also shown by *Orkneyinga Saga* [26] (written c. 1200), which begins with a story about a mythical king Þorri, who held a sacrificial feast at every Midwinter, and his daughter Góa, for which he held a feast a month later; according to the story, the months got their names from these.

See Beckman [5, pp. 32–34] for more details, including a few examples (Þorri, Góa, Einmánuðr, Tvimánuðr) from the Icelandic Sagas. (However, Beckman [5] misplaces the names of the three last summer months in *Edda* [32].)

Dating by giving Icelandic month and day, for example 1 Harpa, has never been used in Iceland. (This is claimed very strongly in Björnsson [10, for example p. 276] and the comment to it by Briem [10, p. 303].) In the Icelandic Almanac (published from 1837 by the University of Copenhagen) there was until 1922 a column giving the days in this reckoning; this was removed when the almanac in 1923 began being published in Iceland [31, pp. 346–347]. We will nevertheless use this form of dates sometimes in the discussions when we find it convenient and helpful, but the reader should remember that it is not standard usage.

2.4. **Further remarks.**

(i) The day was in Iceland in the Middle Ages reckoned from sunrise during summer and from dawn during winter (when the sun rises late in Iceland) [31, p. 316]. That it begins in the morning is also shown by *Bókarbót* "Day comes before night throughout the Icelandic calendar."[27] and *Grágás* "Throughout the calendar a day precedes a night."[28].

I do not know when this was replaced by the modern reckoning from midnight.

(ii) In the medieval texts, periods of several days are often described as 'nights' (*nætur*) rather than 'days' (*dagar*); this is seen in several of the quotes in the present paper.

(iii) A year of exactly 52 weeks with intercalations of a leap week is a rather unique form of calendar, and seems to be an Icelandic invention, but there are some parallels.

The Essenes, a Jewish group (sect) in Qumran (and perhaps elsewhere in Palestine) c. 200 BC – c. 70 AD (now famous for the remnants of their library

[26]Lent always begins in Þorri or Gói. Note that only the beginning of Lent, and not for example the date of Easter Day, is given using Icelandic months in the Easter table [1, pp. 69–70]. I do not know whether the use of Þorri and Gói to specify the beginning of Lent is a reason for the more frequent use of these two month names, or whether, conversely, these two months had a special position already from earlier heathen times, and therefore were used also in learned discussions and Easter tables.

[27]Dagr kemr fyr en nott allz misseris tals islenzcs. [1, p. 78]

[28]Dagr scal fyr koma allz misseris tals en nott. [18, §19 p. 37]

found as the Dead Sea Scrolls) had likewise a calendar of 364 days, with the year always beginning on the same day of week (a Wednesday in their case). They also had months of 30 days, but a different arrangement with 3 months followed by 1 extra day in each quarter, see further Beckwirth [6] and VanderKam [33]. There is no evidence that they used any intercalation at all. (Many intercalation schemes have been proposed by modern researchers, see for example VanderKam [33], but Beckwirth [6] concludes that the Essenes did not use them, nor want them.)

A kind of modern parallel to the Icelandic year is the method of specifying dates (in the standard Gregorian calendar) by day of week and number of week, numbering the weeks by $1, \dots, 52$ or 53 from the beginning of the year. This is standardized by the International Organization for Standardization (ISO) [20], see also Dershowitz and Reingold [12, Chapter 5], and can be interpreted as a year beginning on the Monday closest to 1 January, with weeks beginning on Mondays.[29]

Calendars with 12 months of 30 days each plus some (5 or in leap year 6) extra days have been used by several other people at various times, for example in ancient Egypt from the 3rd millennium BC until several centuries after the Roman conquest, and until today by the Coptic church and in Ethiopia [12, Section 1.5 and Chapter 4]. (A later, European, example is the French revolutionary calendar used 1793–1805.) It seems unlikely that any of these calendars has inspired the Icelandic year with months of 30 days, which therefore seems to be an independent Icelandic reinvention.

3. History

3.1. **The beginnings.** The Icelandic calendar was probably introduced in connection with the establishment in 930 of the *Althingi* (Icel. *Alþingi*), the yearly national assembly that functioned as parliament and court in the Icelandic Commonwealth; see for example Brate [11, p. 20], Þorkelsson [39] and Vilhjálmsson [35].[30] According to Ari hinn fróði's *Íslendingabók* [3, 19]

[29]This method of specifying dates is very popular here in Sweden for schedules at for example school and work, although I have never seen the all-number ISO standard of the type 2009-W27-4 used.

[30]There are, as far as I know, no documents or other contemporary evidence of the use of the Icelandic calendar before the 12th century. (There was essentially nothing at all written in Iceland earlier.) From the 12th century, on the other hand, there is, apart from other sources including the account in *Íslendingabók* [3, 19], a detailed Icelandic book *Rím I* [1, pp. 3–64] on time-reckoning. (Sometimes called *Rímbegla* [1, p. 3], [5, Note 10].) This seems to be a text-book for priests [1, p. XXIII]. (The Icelandic calendar is only a minor part of it; it treats also the Julian calendar, the determination of Easter and astronomy.) There is also a similar work *Rím II* [1, pp. 83–178] from the 13th century, which repeats much of the material in the same or similar formulations. (Both books are edited in [1], together with some related works. We usually give references to only one of parallel formulations in *Rím I* and *Rím II*.) *Rím I* and *Rím II* give a detailed description of the final version of the Icelandic calendar, see the next subsection, but do not mention its history or earlier versions.

(written c. 1125), the Icelandic calendar originally consisted of a year of 364 days (= 52 weeks) without intercalation: "The wisest men in this land had reckoned 364 days in two *misseri*, that is 52 weeks, or 12 months, each of 30 nights, and 4 days in addition. At that time it was, that men noticed by the course of the sun that summer was moving backwards into spring. But no one could tell them that there was one day more in two *misseri* than squared with the number of whole weeks, and that that was the cause."[31]

The year was thus seen to be too short, and on the suggestion of a man called Þorsteinn surtr ("black"), it was decided to try to add a week every seventh year. This reform happened c. 955–960.[32] See further [37]. The central passage about Þorsteinn surtr's calendar reform in *Íslendingabók* [3] (also quoted in [1, pp. 65–66]) is: "he made the motion at the Law Rock that one week should be added every seventh summer, and a trial made how that would answer ... all men were favourably impressed by it, and then it was at once made law"[33]. The reform made the average year 365 days, which we know still is too short. Beckman [1, p. VIII] stresses that the reform is described as an experiment, so it would presumably be modified when it turned out to be insufficient.

Ari hinn fróði too knew that 365 days is too short, but his discussion in [3] seems partly confused when he writes "By right reckoning there are 365 days in every year if it be not leap year; in that case there is one day more, but according to our counting there are 364 days. Now when by our count one week is added to every seventh year, but nothing added by the other count, then seven years will be of equal length by both counts. But if two leap years fall between the years which have to be added to, then the addition must be made to the sixth year."[34] This statement by Ari has been much discussed, in particular the end of it, which seems to describe a modification of Þorsteinn surtr's rule, with intervals of six or seven years between the leap weeks. The same rule is also given, in a slightly different formulation, in the somewhat later *Bókarbót*: "In this reckoning one has to add a week every

[31]Þat vas oc þa es ener Spocosto menn a landi her hofþo taliþ itveim misserom fiora daga ens fiorþa hundraþs. þat verþa vicur ii ens setta tegar en monoþr xii. þritøgnattar oc dagar iiii. umbfram. þa mercþo þeir at Solargangi. at sumar et munaþi aptr til vars ens en þat cunne enꝺi segia þeim at degi einom vas fleira en heilom vicom gegndi i tveim misserom oc þat olli. [3, Ch. IV]

[32]According to [3], which however does not give an exact year for this event. Þorsteinn surtr drowned in 960. See [31, p. 323], [5, p. 26], [39, p. 48].

[33]þa leitaþi hann þess ráþs at Logbergi at et siaunda hvert sumar scylldi auka vicu oc freista hve þa hlyddi. ... aller menn viþ þat vel. oc vas þa þat þegar i Log leidt [3, Ch. IV]

[34]At retto tali ero i hverio are v. dagar ens fiorþa hunþraþs ef eigi es Hlaup ár. en þa einom fleira. Enn at ǫro tali verþa iiii. En þa es aycsc at ǫro tali et siaunda hvert ár vico en øngo at hinu þa verþa vii. ǫr saman iamn loꝺ at hvorotveggia. En ef hlaupǫr verþa ii. a miþli þeira es auka scal þa þarf auca et setta. [3, Ch. IV]

seventh summer. But if there are two leap years in this period, one should increase the sixth."[35]

It is difficult to believe this rule literally. Intervals of six and seven years can never give the same average length of the year as the Julian calendar, and as an improvement of the original rule with seven years intervals, such a rule would be worse than a simpler rule with regular six year intervals. Moreover, the final version of the calendar that we know from for example *Rím I* has intervals of 5 and 6 years with 5 leap weeks in 28 years, see Sections 3.2 and 5, in particular Table 5; it is not known if this version was used during Ari's time, but it was used a few decades later (probably already 1140, see footnote 38), and certainly when *Bókarbót* was written.

Most authors believe that the rule is a correct (but perhaps incomplete) description of the calendar (either the final version or an earlier version), and that "seventh" and "sixth" here should be interpreted in the inclusive sense, that is, as "sixth" and "fifth" (this method of counting is common in Latin, but not in Icelandic), see for example Jónsson [21], Björnsson [10], Þorkelsson [38, 39], Einarsson [14, 15], Benediktsson [19], Vilhjálmsson [34, 35]. An exception is Beckman [5, pp. 22–28], who argues that Ari uses the expressions in the same sense as we do, and that the comment is incorrect. (Einarsson [14, 15] believes that Ari consistently used inclusive counting, also in the description of Þorsteinn surtr's reform, which thus added a leap week every *sixth* year. This would give an average year of $365\frac{1}{6}$ days, only 2 hours less that the Julian year (1 day in 12 years); the difference from the tropical year is slightly less, about 1 day in 13 years. However, there are other problems with this, and most authors assume that Ari used inclusive counting only in the last part of the chapter; see Vilhjálmsson [34] for a discussion.)

Even with the assumption of inclusive counting, the rule is not crystal clear, and several attempts of interpretations have been made.

Björnsson [10] tries to use the rule to reconstruct a version "Aratal" of the calendar used in Ari's time, slightly different from the later version, but Þorkelsson [38] points out that Björnsson's reconstruction is inconsistent; see Beckman [5] for further criticism.

Þorkelsson [39] proposes an interpretation that fits the final version of the calendar (see Table 5). His interpretation is as follows, starting with a year with *sumarauki*, which we number as year 1: It does not matter whether year 1 is a Julian leap year or not, since the leap day (24 or 25 February according to medieval reckoning) comes before the *sumarauki*. If there is only one leap year in years 2–7, then the next *sumarauki* is in year 7. If there are two leap years in years 2–7, then the next *sumarauki* is in year 6; this includes the case when the leap years are years 3 and 7, and in this case year 7, which thus is the year after the next *sumarauki*, is regarded as

[35] I þvisa tali þarf auca et siaunda hvert svmar vicu. En ef hlaupar verþa tvau a þeiri stund, þa scal auca it seta. [1, p. 78]

used and not counted when finding the following *sumarauki*. See also [22, Interkalation].

Note that any leap week rule that yields the same average length of the year as the Julian calendar must have 5 leap weeks in 28 years (on the average, at least); the natural arrangement has 3 gaps of 6 years and 2 of 5 years in some pattern that is repeated cyclically. However, with a rule of Ari's type, the distance from one leap week to the next depends only on the number of years until the next Julian leap year; since there are only 4 possibilities, the rule leads to a pattern that repeats after at most 4 leap weeks – in other words, the desired cycle with 5 leap weeks is impossible. The proposal by Þorkelsson [39] just described avoids this problem by the added rule that the next leap year may be "used"; this effectively introduces a fifth possibility (one year before a Julian leap year that is used), and enables a cycle of 5 leap weeks.

Further reforms (one or several) of the intercalation rule must have occurred after 960, both in order to follow the seasons (that is, the tropical year), and because Iceland became Christian in 1000 (or 999 [13]) and then the Catholic church arrived using the Julian calendar. Nothing is known about the detailed operation of the Icelandic calendar in this period; presumably leap weeks were inserted when needed, but we do not know according to which rules, if any. (The Althingi was moved from 9 weeks after the First Day of Summer to 10 weeks after it in 999 [3, Ch. VII], which may indicate that the intercalations so far had been insufficient. As many have observed, this fits well with the fact that if leap weeks had been added every seventh year since the reform c. 960, then the calendar would have drifted about 10 days during these 40 years.) The law *Grágás* [18] (written down in the 13th century but presumably showing older practice) says, in the section on the duties of the Lawspeaker: "The Lawspeaker has to announce ... and the calendar, and also if men are to come to the Althingi before 10 weeks of summer have passed, and rehearse the observance of Ember days and the beginning of Lent, and he is to say all this at the close of the Althingi."[36] This does not exclude the possibility that the Lawspeaker used some fixed rules for the intercalation, but it also allows the possibility that intercalation was done on an ad hoc basis. See also Vilhjálmsson [35] for a discussion of Icelandic astronomical knowledge at this time.

The Icelandic calendar presumably had some roots in earlier Scandinavian time-reckoning, see Brate [11], Ginzel [17, §§228–230] and Schroeter [31, pp. 300–315]. However, only a little is known about the details of earlier time-reckoning, so it is difficult to say how much of the Icelandic calendar that really was common Scandinavian. Some features, such as the year of 364 days, seem to be uniquely Icelandic. The partition of the year into summer

[36]Lögsogo maðr a up at segia ... oc misseris tal. oc sva þat ef menn scolo coma fýr til alþingis en x. vicor ero af sumre. oc tina imbro daga halld. oc fösto iganga. oc skal hann þetta allt mæla at þinglavsnom. [18, §116 p. 209]

and winter half-years was old Germanic [17, p. 58]. (For example, it existed some centuries earlier in the Anglo-Saxon calendar according to Bede [7, §15], written c. 725 but here reporting an earlier time.) This division into summer and winter existed in Norway and Sweden too, but there the beginnings were fixed to 14 April and 14 October and not to a specific day of week, at least as far back as there is any documentation [11, pp. 26, 28], [5, pp. 20–21, 38]; this is for example shown in *Rím II*: "On Calixtus [14 October] comes winter according to Norwegian reckoning, and on Tiburtius [14 April] comes summer."[37].

Beckman [1, pp. CXLVII–CLVI] points out that the Lapps also used a time-reckoning based on weeks (with 13 months of 4 weeks each), which presumably has the same origins.

It seems clear that the Icelanders originally, as other Scandinavians, used lunar months, but not much is known about details [11, 17, 5]. Note that the Icelandic month names Þorri and Góa existed also in the other Scandinavian countries [11, pp. 26–27], and certainly are from before the Christian time. (The Anglo-Saxon names for their old lunar months, given by Bede [7, §15] c. 725, are quite different from the Icelandic, however.) As a relic of this, the Icelandic Almanac [2, 28] gives traditional names for the Moon for 6 lunar months (reckoned from New Moon) each year: *jólatungl*, *þorratungl*, *góutungl*, *páskatungl*, *sumartungl*, *vetrartungl*. Of these, *páskatungl* is the Paschal Moon (central for the determination of Easter in the Christian calendar, although these calculations do not use the actual, astronomical Moon), and *þorratungl* and *góutungl* are the names for the two preceding lunar months. See further [28] and [31, p. 336].

Þorkelsson [39, pp. 53–55] suggests that the lunar months were impractical during the summer in Iceland because then there is no real night and the full moon is very low in the sky at these latitudes; hence a week reckoning replaced the lunar months during the summer, while the winter months were kept and only later, and not completely, were replaced by the week reckoning. This would explain why the names of the winter months are used much more than the summer months as discussed in Section 2.3. At some time during this process (probably c. 930 as said above), the calendar was reorganized and the months were fixed to 30 days and defined by the week reckoning instead of the moon.

3.2. **Linked to the Julian calendar.** At some time during the 11th or 12th century the calendar was linked to the Julian calendar by adopting its mean length of $365\frac{1}{4}$ days per year; this was effected by the intercalation of 5 leap weeks in 28 years, and the calendars were linked so that the First

[37] Calixtus messa kemur vetur at norénu tali. enn Tiburcius messo sumar. [1, p. 156 §136]

Day of Summer always fell in the week 9–15 April.[38] [39] This is the version described in *Rím I* [1] from the late 12th century, and a century later in *Rím II* [1]. (See also the comments by Beckman [1, Inledning] and [5], Björnsson [10], Þorkelsson [36, 39], Schroeter [31].)

The church used the Julian calendar, but the Icelandic calendar, combined with Christian holidays and saints' days (determined by the Julian calendar), continued to be the main calendar for civil use for centuries. Dates were usually given using holidays and saints' days, but also often using the Icelandic calendar and its counting of weeks; see further Beckman [5, pp. 36–38].

Remark 1. *Rím I* [1] says "Summer shall not begin before 14 days after the Annunciation of Virgin Mary and not later than 21 days after, and the first day is a Thursday."[40] Since the Annunciation is 25 March, this specifies the interval 8–15 April for the First Day of Summer. However, it is clear from other evidence that the interval really was 9–15 April. This is explicitly stated in *Rím II* [1] (13th century): "Summer shall not come closer to the Annunciation of Virgin Mary than 15 days after it and not later than 21, and a Thurday shall be the first day in the summer"[41]. Moreover, a First Day of Summer on 8 April is incompatible with other statements in *Rím I*: the preceding winter or summer would violate the discussion of the beginning of Lent in *Rím I* [1, pp. 25–28 §§30–34] (at least assuming the explicit rule on *rímspillir* in *Rím I*), see Appendix C.1, as well as one or several of the claims quoted in Appendix D. See further Beckman [1, p. LXXI] and [5, p. 27].

It is, as suggested by Beckman [1, p. LXXI], possible that the statement in *Rím I* [1, p. 22 §26] yielding the range 8–15 April is an earlier rule (either an informal rule or a formal law) which was used during some period but did not specify the calendar completely.

[38]The 12th century Easter table [1, pp. 69–70] covering the years 1140–1195 (two solar cycles, see Appendix A.3), shows (apart from standard international information) the week in Þorri or Góa in the Icelandic calendar when Lent begins (see Appendix C.1), and it marks the years with *sumarauki*. All is in accordance with the rules described in this paper (apart from an error for Lent 1193, where G[óa] is written instead of Þ[orri]; this must be a typo, medieval or in [1]), which is strong evidence that the rules were in effect at the latest in the middle of the 12th century.

[39]The comment on the intercalation in *Íslendingabók* [3] (c. 1125), see Section 3.1 above, is taken as evidence by for example Björnsson [10] and Beckman [1, p. VIII–IX] that Ari hinn fróði did not know the later rules, and thus the final reform linking the calendar to the Julian one was made later; this gives a dating to the middle of the 12th century. (Beckman [5, p. 28] guesses between 1140 and 1173.) However, since Beckman [5] dismisses Ari´s comment as inaccurate, it seems inconsistent by him to draw this conclusion from it. On the other hand, if for example the interpretation by Þorkelsson [39] (see again Section 3.1) is correct, then the rules are probably older than *Íslendingabók* (although as discussed by Þorkelsson [39], a later final adjustment also is possible).

[40]Skal sumar koma eigi nér Mariu messo um fostu helldr enn XIIII nottum efter ok eigi firr enn einne nott ok XX, ok skal enn fimte dagr viko vera fyrstur i sumri. [1, p. 22 §26]

[41]Sumar skal eigi nær koma Mariu mesu of faustu enn 15 nottum epter ok eigi fyr enn 21, ok skal v. dagr vera fystur i sumri [1, p. 83 §3]

Remark 2. The medieval texts *Rím I* and *Rím II* do not relate the Icelandic calendar directly to the Julian; as in the quotes in Remark 1, the Icelandic calendar is defined using the ecclesiastical calendar with saints' days. Similarly, *Rím II* [1, pp. 168–170 §§160–162] gives the first day of nine of the months, in complete agreement with Table 1 but stated using saints' days. (A partial exception is the following, where a double dating is used for clarity: "There shall be one Tuesday between the Tuesday that is first in Einmánuðr, and that day that is 6 days after St Matthias [24 February], but 7 days if it is a leap year, and that is the second of March."[42]) It is interesting to note that *Rím I* begins by defining the Julian months in terms of the ecclesiastical calendar "September ... comes a week before the Nativity of Blessed Virgin Mary, ... October ... comes 2 days after Michaelmas, ... November comes on All Saints' Day ..."[43], and not the other way round as we would do today; this too shows that the saints' days were more well-known that the Julian months.

3.3. **Linked to the Gregorian calendar.** When the Julian calendar was replaced by the Gregorian in 1700, the Icelandic calendar became instead adapted to the Gregorian one, and the First Day of Summer was fixed to be the Thursday in 19–25 April [28, 31], which has remained the rule until present. See Section 3.4 for details of the transition.

The general population continued to use the Icelandic calendar until the late 18th century, when it was replaced by the Gregorian calendar for general use [9, pp. 8, 14], [28, tímatal, fornískenskt]. (The Icelanders, of course, observed the Christian holidays, which always followed the Julian or Gregorian calendar; however, only the clergy was concerned with the calendar behind the holidays and the calculation of the correct dates for them.) The Icelandic calendar was occasionally used until the early 20th century [9, p. 8].

The First Day of Summer has always been celebrated in Iceland. Although not a Christian holiday, it was even celebrated with mass until 1744, as was (at least in the northern episcopate of Hólar) the First Day of Winter [9, pp. 16, 60–61].

3.4. **The calendar change in 1700.** Iceland changed from the Julian to the Gregorian calendar in November 1700, when Saturday 16 November (Jul.) was followed by Sunday 28 November (Greg.) [23, pp. 1375–1376], [31, p. 333].[44]

[42]Þridiu dagr ein skal vera i milli þess þridiu dags, er fystur er i ein manadi, ok þess dags, er VI nottum er epter Mattias, en VII, ef hlaupar er, en sa er anar i marcio. [1, pp. 169–170 §162]

[43]September ... kemr viku fyrer burdar dagh Mariu, ... october ... kemr II nottum efter Michaels messo, ... November ... kemur allra heilagra messo ... [1, pp. 7–8 §4]

[44]Iceland belonged at that time to Denmark, which changed to the Gregorian calendar in February 1700, when Sunday 18 February (Jul.) was followed by Monday 1 March (Greg.) [17, p. 274]. It seems that the royal ordinance on 28 November 1699 [17], [31, p. 334 n. 1] was made too late to reach Iceland in time since there were no communications

The change to the Gregorian calendar, however, did not affect the Icelandic year, since the date for the First Day of Summer simultaneously was changed by 10 days, from 9–15 to 19–25 April as said above. In particular, there was no discontinuity in 1700 (or in any neighbouring year); the First Day of Summer 1700 was Thursday 11 April (Jul.), which equals Thursday 22 April (Greg.) as it would have been with the Gregorian rule. Neither were any days skipped in the Icelandic calendar that year; the Icelandic year was a regular year with 364 days, and the next First Day of Summer came on Thursday 21 April 1701.

Note that the difference between the Julian and Gregorian calendars was 10 days during the 16th and 17th centuries, and still when Denmark switched in February 1700, but from March 1700 it increased to 11 days since 1700 was a Julian leap year but not a Gregorian. Thus 1700 was a leap year in Iceland, and the difference had increased to 11 days when the Gregorian calendar was adopted in November. Nevertheless, the dates of the First Day of Summer were shifted only 10 days and not 11. This made no difference in 1700, or in 1701, but in 1702 it meant the difference between *sumarauki* or not.

The adaption of the Icelandic calendar to the Gregorian was decided by the Althingi on 1 July 1700 [23, p. 1376].[45] The decision actually states the dates of the First Day of Winter, which at that time was a Friday, see Section 2.1: "The beginning of winter, which until now has been on that Friday that is between the 9th and 18th October, will from now on be on that Friday that is between the 19th and 28th October ... and then is easily reckoned Midwinter, the beginning of Þorri, as is the beginning of summer, after 26 weeks of winter"[46].[47] Note that this formulation really does not specify the First Day of Winter completely, since the range is the 8 days 20–27 October ("between" is taken in the strict sense here), so in some years (when both 20

during the winter; therefore a separate royal ordinance for Iceland (and the Faroe Islands) was issued by the (Danish) king on 10 April 1700 specifying the transition in November [23, pp. 1375–1376].

[45]The king was apparently not interested in the Icelandic calendar, so this was left to the Icelanders themselves.

[46]Um vetrarkomu, sem hingað til hefir verið á þann föstudag, sem inn hefir fallið millum þess 9da og 18da oktobris, vill nú tilreiknast að inn falli hér eftir á þeim föstudegi, sem er á milli þess 19da og 28da oktobris ... og er þá sjálfreiknað um miðsvetrar þorrakomu, svo og um sumarkomu, þá 26 vikur eru af vetri [23, p. 1376]

[47]According to Schroeter [31, p. 334], the original decision by the Althingi made on 1 July 1700 was to shift the dates of the First Day of Winter by 11 days to 21–28 October ("between the 20th and 29th October"); this made no difference in 1701, but it would in 1702 when, however, the First Day of Winter in most places was taken to be Friday 20 October (instead of 27 October as the new rule would give); the Althingi followed this and by a new decision in 1703 adjusted the shift to the formulation quoted above with a shift of 10 days. (The reason is not known for shifting 10 days instead of 11, which meant that there was *sumarauki* in 1703 and not in 1702. Note that the Julian version would have had *sumarauki* in 1702, so the reason cannot be that a conservative population resisted changes – the result was the opposite.)

and 27 October are Fridays) there is an ambiguity. (The stated rule for the Julian calendar, with the range 10–17 October, is similarly incomplete. The ranges in the law for the First Day of Winter in the Julian and Gregorian versions do not agree with the dates in Table 1 since the First Day of Winter here is a Friday, and thus the day before the first of W1 (Gormánuður) in Table 1.) Nevertheless it seems clear that the intended meaning, and the actual implementation, was to shift the dates by 10 but otherwise keep the same rules for leap weeks and *rímspillir*; in particular, the First Day of Summer, which according to the law above comes 26 weeks (minus a day, since it is a Thursday) after the First Day of Winter, is restricted to the range 19–25 April of 7 days (see Table 1), which determines the First Day of Summer and thus the year completely as discussed in Section 4.[48]

The shift by 10 days means that the First Day of Summer (and thus every other day in the Icelandic calendar) on the average falls on the same day in the Gregorian calendar (and thus in the tropical year) as it did in the 16th and 17th centuries. (This is about 3 days later in the tropical year than it was during the 12th and 13th centuries.)

The first time after 1700 that the Julian and Gregorian versions of the Icelandic calendar differ was at Midsummer 1702, when there would have been *sumarauki* in the Julian version, but not in the Gregorian; nine months later, the First Day of Summer was on Thursday 19 April 1703 (Greg.), but would have fallen one week later (on 15 April Jul. = 26 April Greg.) according to the Julian rule. The two versions coincide during the 16th and 17th centuries, when the difference between the Julian and Gregorian calendar was 10 days. Going backwards in time, with the proleptic Gregorian calendar, the last year before 1700 with a difference would have been 1495 (long before the construction of the Gregorian calendar).

3.5. **Present use.** The Icelandic calendar is preserved as a cultural heritage, and the First Day of Summer (*Sumardagurinn fyrsti*) is still a public holiday [25] (and a flag day) in Iceland. The Icelandic Almanac [2] shows besides the standard Gregorian calendar, including the modern numbering of weeks (beginning on Mondays [20]), also the Icelandic months (more precisely, the beginning of each month) and the numbering of summer weeks and winter weeks (beginning on Thursdays and Saturdays) according to the Icelandic calendar.

Some other traditions are also still connected to the Icelandic calendar:

This is discussed in detail by Sæmundsson [30]. He finds the description by Schroeter [31] partly inaccurate and concludes that probably the Althingi decided on a 10 day shift already 1700, but that there was disagreement leading to confusion in 1702 and 1703.

[48]I do not know why the law is stated in terms of the First Day of Winter instead of the First Day of Summer, which would have made the rule unambiguous. Note that a rule for the First Day of Summer implicitly determines whether there is a *sumarauki*, by looking also at the next First Day of Summer. Sæmundsson [30] gives some suggestions for the choice of the First Day of Winter in the law.

The first day of Þorri is called *bóndadagur* (Husband's day) and the last *þorraþrœll*. The first day of Góa is called *konudagur* (Wife's Day) and the last *góuþrœll* [2, 28]. A modern custom (from the 1980's and 1950's) is that wives give their husband flowers on *bóndadagur*, and conversely on *konudagur* [9, pp. 92, 96].

Another tradition is the feast *þorrablót* in Þorri. This is mentioned in the Saga literature, and was revived around 1870 [9, pp. 88–92].

Réttir, the annual round-up of sheep, begins Thursday in the 21th week of summer [28, 9].

It is interesting that among the months, Þorri and Góa seem to have a special position in the general population today as well as 1000 years ago, see Sections 2.3 and 3.1.

4. Calculations and calendar conversions

To convert dates between the Icelandic calendar and another calendar such as the Julian or Gregorian, it suffices to know the dates in the other calendar of the First Day of Summer each year; knowing the First Day of Summer for a given year and the next one, we know the length of the year and thus whether there is a leap week (*sumarauki*) or not, and thus all dates can be translated. For calculations involving Icelandic months, especially computer programs, it may be convenient to count forward from the First Day of Summer for the first three months of the summer (S1–S3 in our notation), including *aukanœtur* and *sumarauki*, and backwards from the First Day of Summer next year for the remaining nine months S4–W6; in this way the fact that only some years have *sumarauki* is taken care of automatically. In particular, the First Day of Winter is always 180 days before the next First Day of Summer.

In the following two sections we give some details on calculations of the First Day of Summer for the versions of the Icelandic calendar tied to the Julian and Gregorian, respectively, and in Appendix B we give further formulas; the interested reader can easily construct complete conversion algorithms based on the formulas given here and algorithms for the Julian and Gregorian calendar in, for example, Dershowitz and Reingold [12] (which is an excellent source for many details on calculations and conversions for many different calendars). The details can, of course, be varied, and we give several versions, while other variants are left to the reader. Some similar formulas can be found in [10].[49] [50] We give also several tables showing the results of the calculations; similar tables can be found in for example Bilfinger [8], Beckman [1, Inledning], Björnsson [10] and Schroeter [31].

[49] Computer programs are given in *Almanak Háskólans* 1986 and 1991.

[50] A traditional method to calculate the calendar (both Gregorian and Icelandic) was to count on the fingers. This was described by bishop Jón Árnason [4] in 1739; for a modern version see [29].

4.1. ***Rímspillir.*** The date in the Julian or Gregorian calendar of the start of the months S4–W6 can, as said above, be found by counting backwards from the First Day of Summer next year, with 30 days per month. Note that for S4–W5, the result depends not only on the date of the First Day of Summer next year, but also on whether it falls in a Julian or Gregorian leap year (having 29 February) or not. Hence, the dates for the beginning of one of these months vary over a range of 8 days, as shown in Table 1, see also Tables 5 and 7, while the beginning of one of the months S1–S3 and W6 varies only over 7 days. In particlar, this means that the beginning of one of the latter 4 months in a given year can be determined as the unique day with the correct day of week in the interval given in Table 1, while more information sometimes is needed for the months S4–W5.

It is easy to see that, for any of the months S4–W5, the last of the 8 possible days occurs only when the First Day of Summer next year is on the latest possible date (15 April or 25 April), and further falls in a (Julian or Gregorian) leap year. This exceptional case is called *rímspillir* (= "calendar destroyer") or *varnaðarár* = "warning year". (So already in the 12th century manuscript *Rím I* [1].)

Rím II is very precise: "*Varnaðarár* begins at Midsummer the 8th year in the solar cycle and ends at the leap day the 9th year in the solar cycle."[51]

It is easily seen that when *rímspillir* occurs, the First Day of Summer is on the *second* possible date (10 April [Jul.] or 20 April [Greg.]); conversely *rímspillir* occurs every time the First Day of Summer is on this date and the next Julian or Gregorian year is a leap year.

Note the other (non-*rímspillir*) instances of *sumarauki* occur precisely in the years where the First Day of Summer is on the earliest possible date (9 April [Jul.] or 19 April [Greg.]).

When *rímspillir* occurs, the First Day of Winter and the winter months come a week later than in another year with the same dominical letter. The medieval Icelanders viewed this the other way round, seeing the Icelandic calendar as the fixed reference point: "and are all mass days a week earlier in the calendar, than if there were no *rímspillir*"[52].

4.2. **Notation.** In some formulas it will be convenient to code days of week by numbers. We choose to use the numbering 1 (Sunday) to 7 (Saturday), in accordance with the Icelandic names of some of the days of week (and the traditional Judaeo-Christian reckoning that the names are based on), see Section 2.2 and Table 3. Note that [12] uses a different convention, and ISO [20] yet another; the reader that desires one of these numberings can easily modify our formulas.

[51] Varnadar ar hefst at midiu sumri hin atta vetr i solar olld ok lykz æa hlaupars deigi hin 9. vetur i solar olld. [1, p. 136 §104]

[52] ok verda aller messo daghar viko fyrr i misseris taleno, helldr enn þa ef eigi vére rimspillerenn [1, *Rím I* p. 24 §28]

We let $m \bmod n$ denote the remainder when m is divided by n; this is an integer in the range $0, \dots, n-1$. (Here m and n are integers. We only consider $n > 0$, but m may be of any sign; care has to be taken with this in computer implementations.)

Similarly (following [12]), we let $m \operatorname{amod} n$ denote the remainder adjusted to the range $1, \dots, n$. This means that $m \operatorname{amod} n = m \bmod n$ except when m is a multiple of n; then $m \bmod n = 0$ and $m \operatorname{amod} n = n$. Equivalently, $m \operatorname{amod} n = \big((m-1) \bmod n\big) + 1$.

We use the standard notation $\lfloor x \rfloor$ for the integer satisfying $x-1 < \lfloor x \rfloor \le x$, that is x rounded down to an integer.

5. The Julian version (12th c. – 1700)

During the time the Icelandic calendar was tied to the Julian (12th century – 1700), the First Day of Summer was the Thursday falling in 9–15 April, that is, the first Thursday on or after 9 April.

The Julian calendar has a cycle of 28 years (known as the *solar cycle*) for the days of week, see Appendix A.3. Hence the date of the First Day of Summer has a cycle of 28 years, and consequently so has the complete Icelandic calendar (Julian version). In 28 Julian years there are

$$28 \cdot 365 + 7 \text{ days} = 10227 \text{ days} = 1461 \text{ weeks} = 28 \cdot 52 + 5 \text{ weeks}.$$

Hence, in each period of 28 Icelandic years, there are 5 leap weeks (*sumaraukar*); in other words, there are 23 ordinary Icelandic years of 364 days (52 weeks) and 5 Icelandic leap years of 371 days (53 weeks). The leap years are evenly spread out with distances of 5 or 6 years (that is, 4 or 5 ordinary years in between); the average gap is $28/5 = 5.6$ years, with 2 gaps of 5 years and 3 gaps of 6 years in each cycle. The entire cycle will be given in Table 5.

We give some ways to calculate the First Day of Summer (Julian version) for a given year.

5.1. **Using a day-of-week function.** Assume that $\mathbf{dowJ}(d, m, y)$ is a function giving the day of week (as a number 1–7, see above) of day d, month m, year y in the Julian calendar. (As said above, see for example [12] for the construction of such a function.) Thursday is day 5, and thus the date, in April, of the First Day of Summer in Julian year y can be written

$$9 + \big((5 - \mathbf{dowJ}(9, 4, y)) \bmod 7\big) = 9 + \big((12 - \mathbf{dowJ}(9, 4, y)) \bmod 7\big) \quad (5.1)$$

(where the second version avoids negative numbers in the calculation).

5.2. **Using dominical letters.** Since the First Day of Summer is a Thursday, the day 3 days later (4 Harpa) is a Sunday in 12–18 April. These days have calendar letters (see Appendix A.1) DEFGABC, respectively, which gives the following table connecting the dominical letter and the First Day of Summer of a Julian year. A Julian leap year has two dominical letters; it is the second one (valid for March–December) that is used here.

Dominical letter	A	B	C	D	E	F	G
First Day of Summer	13	14	15	9	10	11	12

TABLE 4. Dominical letters and First Day of Summer in the Julian version of the Icelandic calendar

To express this relation in a formula, let δ be the number corresponding to the dominical letter (A=1, B=2, ..., G=7); then the First Day of Summer is

$$\big((\delta+3) \mod 7\big)+9 \quad \text{April}. \tag{5.2}$$

By the discussion in Section 4.1 and Table 4, the Icelandic year is a leap year, that is a leap week (*sumarauki*) is added, in the following cases [31]:

- The First Day of Summer is on 9 April. Equivalently, the dominical letter is D (or ED).
- The First Day of Summer is on 10 April and the next year is a Julian leap year. Equivalently, the dominical letter is E and the next year has dominical letters DC. (*Rímspillir*.)

This rule is stated already in *Rím II* [1, p. 144 §133], in the form that a week is added if the dominical letter is D, and *Jónsmessa* (St John Baptist, 24 June) is on a Wednesday, and also in year 8 of the solar cycle when the dominical letter is E and *Jónsmessa* is on a Tuesday. (The conditions on St John Baptist are equivalent to the conditions on the dominical letter, since 24 June has calendar letter G, cf. Appendix A.1.)

5.3. **Using concurrents.** The concurrent is an alternative to dominical letters, see Appendix A.2. By (A.1), we can translate (5.2): If κ is the concurrent a given year, then the First Day of Summer is

$$\big((10-\kappa) \mod 7\big)+9 \quad \text{April}. \tag{5.3}$$

(This also easily follows from (5.1).)

Using (A.2), it follows that the First Day of Summer year y AD is

$$15-\big((y+\lfloor y/4\rfloor) \mod 7\big) \quad \text{April}. \tag{5.4}$$

5.4. **Using Julian day numbers.** Assume that $\mathbf{JDJ}(d,m,y)$ is a function giving the Julian day number (see Appendix A.5) of day d, month m, year y in the Julian calendar. (See for example [12] or [16] for the construction of such a function.) By (A.6),

$$\begin{aligned}\mathbf{dowJ}(d,m,y) &= (2+\mathbf{JDJ}(d,m,y)) \bmod 7 \\ &= \big((1+\mathbf{JDJ}(d,m,y)) \mod 7\big)+1,\end{aligned} \tag{5.5}$$

Sol. cyc.			S1 Apr	S2 May	S3 Jun	S4 Jul	S5 Aug	S6 Sep	W1 Oct	W2 Nov	W3 Dec	W4 Jan	W5 Feb	W6 Mar
1	GF		**11**	11	10	14	13	12	**12**	11	11	10	9	11
2	E		**10**	10	9	13	12	11	**11**	10	10	9	8	10
3	D	∗	**9**	9	8	19	18	17	**17**	16	16	15	14	16
4	C		**15**	15	14	18	17	16	**16**	15	15	14	13	14
5	BA		**13**	13	12	16	15	14	**14**	13	13	12	11	13
6	G		**12**	12	11	15	14	13	**13**	12	12	11	10	12
7	F		**11**	11	10	14	13	12	**12**	11	11	10	9	11
8	E	∗	**10**	10	9	*20*	*19*	*18*	***18***	*17*	*17*	*16*	*15*	16
9	DC		**15**	15	14	18	17	16	**16**	15	15	14	13	15
10	B		**14**	14	13	17	16	15	**15**	14	14	13	12	14
11	A		**13**	13	12	16	15	14	**14**	13	13	12	11	13
12	G		**12**	12	11	15	14	13	**13**	12	12	11	10	11
13	FE		**10**	10	9	13	12	11	**11**	10	10	9	8	10
14	D	∗	**9**	9	8	19	18	17	**17**	16	16	15	14	16
15	C		**15**	15	14	18	17	16	**16**	15	15	14	13	15
16	B		**14**	14	13	17	16	15	**15**	14	14	13	12	13
17	AG		**12**	12	11	15	14	13	**13**	12	12	11	10	12
18	F		**11**	11	10	14	13	12	**12**	11	11	10	9	11
19	E		**10**	10	9	13	12	11	**11**	10	10	9	8	10
20	D	∗	**9**	9	8	19	18	17	**17**	16	16	15	14	15
21	CB		**14**	14	13	17	16	15	**15**	14	14	13	12	14
22	A		**13**	13	12	16	15	14	**14**	13	13	12	11	13
23	G		**12**	12	11	15	14	13	**13**	12	12	11	10	12
24	F		**11**	11	10	14	13	12	**12**	11	11	10	9	10
25	ED	∗	**9**	9	8	19	18	17	**17**	16	16	15	14	16
26	C		**15**	15	14	18	17	16	**16**	15	15	14	13	15
27	B		**14**	14	13	17	16	15	**15**	14	14	13	12	14
28	A		**13**	13	12	16	15	14	**14**	13	13	12	11	12

TABLE 5. Solar cycle and beginning of months in the Julian version of the Icelandic calendar. The second column shows the dominical letter(s). (These columns are valid only until Dec. 31, and therefore not for the months W4–W6, which are in the next Julian year.) A ∗ in the third column marks Icelandic leap years with *sumarauki*. The first days of summer and winter are marked by boldface, and *rímspillir* is marked by italics (line 8).

and thus, by (5.1), the Julian day number of the First Day of Summer in Julian year y can be written

$$\begin{aligned} &\mathbf{JDJ}(9,4,y) + \big((3 - \mathbf{JDJ}(9,4,y)) \mod 7\big) \\ &= \mathbf{JDJ}(15,4,y) - \big((3 + \mathbf{JDJ}(9,4,y)) \mod 7\big) \\ &= \left\lfloor \frac{\mathbf{JDJ}(12,4,y)}{7} \right\rfloor \cdot 7 + 3. \end{aligned} \tag{5.6}$$

See further Appendix B.

5.5. **The solar cycle.** As said above, and in more detail in Appendix A.3, the solar cycle is a cycle of 28 years in the Julian calendar for the days of week, and thus of the dominical letters. The place of a year in the solar cycle thus determines the dominical letter and hence the First Day of Summer; since the solar cycle also determines the dominical letter and the First Day of Summer next year, the entire Icelandic year is determined by the place in the solar cycle. A list of the 28 years in the cycle is easily made, for example using Table 4 or (5.1); the result is given in Table 5. The position of a given year is found by (A.4); for example, the last Julian year on Iceland, 1699, has number 28 in the solar cycle.

From Table 5 we immediately see that the Icelandic leap years are the years 3, 8, 14, 20, 25 in the solar cycle, with *rímspillir* year 8. (This has been noted from the early days of the calendar. These years are marked with *wiþl* (*wiþ lagning* = "addition" = *sumarauki*) already in the Easter table [1, pp. 69–70] for the two solar cycles 1140–1195. *Rím II* says "shall be added to the summer 5 times in the solar cycle; the first time when the dominical letter is D, the second time when it is E, and then three times when it is D"[53]. *Rím I* says "... *rímspillir*, which is that year that is eighth in the solar cycle"[54].)

Rímspillir thus occurs once every 28 years, in year 8 in the solar cycle. From (A.4) follows that the years with *rímspillir* begin in the Julian years that give the remainder 27 when divided by 28. In the 12th–17th centuries, these are the years 1119, 1147, 1175, 1203, 1231, 1259, 1287, 1315, 1343, 1371, 1399, 1427, 1455, 1483, 1511, 1539, 1567, 1595, 1623, 1651, 1679. (But recall that we do not know exactly when the final version of the Icelandic leap year rule was introduced.) As Beckman [1, 5] points out, a simple rule is that *rímspillir* ends in the Julian years that are divisible by 28.

6. The Gregorian version (1700 – present)

Since the Icelandic calendar became tied to the Gregorian in 1700, the First Day of Summer has been the Thursday falling in 19–25 April, that is, the first Thursday on or after 19 April.

The Gregorian calendar has a cycle of 400 years for the leap years; this cycle contains 97 leap years and thus $400 \cdot 365 + 97 = 146097$ days, which happens to be divisible by 7 and thus a whole number of weeks. The leap year cycle of 400 years is thus also a cycle for the days of week, and hence for the Icelandic calendar (Gregorian version). In 400 Gregorian years there are

$$400 \cdot 365 + 97 \text{ days} = 146097 \text{ days} = 20871 \text{ weeks} = 400 \cdot 52 + 71 \text{ weeks}.$$

[53] ok skal vid sumar leggia 5 sinnum i solar aulld: hit fysta sin, er drottins dagur er æa d, anat sin, er han er æa e, þa þryssvar sinum þadan af, er hann er æa d [1, p. 128 §86]

[54] ... rimspillerenn, er sa vetur enn atte i concurrentes aulld hverre [1, p. 24 §28]

Hence, in each period of 400 Icelandic years, there are 71 leap weeks (*sumaraukar*); in other words, there are 329 ordinary Icelandic years of 364 days (52 weeks) and 71 Icelandic leap years of 371 days (53 weeks). The leap years are rather evenly spread out with distances of 5 or 6 years (that is, 4 or 5 ordinary years in between); except that (because of the Gregorian leap rule exceptions for years divisible by 100), once in the cycle there is a distance of 7 years. (This happened 1696–1703, straddling the introduction of the Gregorian version of the Icelandic calendar, and will happen next time 2096–2103.) The average gap is $400/71 \approx 5.6338$ years, with 27 gaps of 5 years, 43 gaps of 6 years and 1 gap of 7 years in each cycle.

We give some ways to calculate the First Day of Summer (Gregorian version) for a given year. The full pattern is given in Table 7. (Table 7 gives also the beginning of each month. We ignore in this section, including the table, the different placement of the leap week in the almanacs until 1928, see Section 7.1, which affects S4–S6 in Icelandic leap years. We also ignore the beginning of winter on a Friday before 1837, see Section 2.1.)

6.1. **Using a day-of-week function.** Assume that $\mathbf{dowG}(d, m, y)$ is a function giving the day of week (as a number 1–7, see above) of day d, month m, year y in the Gregorian calendar. (Again, see [12] for the construction of such a function.) Thursday is day 5, and thus the date, in April, of the First Day of Summer in Gregorian year y can be written

$$19 + \big((5 - \mathbf{dowG}(19, 4, y)) \mod 7\big) = 19 + \big((12 - \mathbf{dowG}(19, 4, y)) \mod 7\big). \tag{6.1}$$

6.2. **Using dominical letters.** Since the First Day of Summer is a Thursday, the day 3 days later (4 Harpa) is a Sunday in 22–28 April. These days have calendar letters (see Appendix A.1) GABCDEF, respectively, which gives the following table connecting the dominical letter and the First Day of Summer of a Gregorian year. A Gregorian leap year has two dominical letters; it is the second one (valid for March–December) that is used here.

Dominical letter	A	B	C	D	E	F	G
First Day of Summer	20	21	22	23	24	25	19

TABLE 6. Dominical letters and First Day of Summer in the Gregorian version of the Icelandic calendar

To express this relation in a formula, let δ be the number corresponding to the dominical letter (A=1, B=2, ..., G=7); then the First Day of Summer is

$$(\delta \mod 7) + 19 \quad \text{April}. \tag{6.2}$$

By the discussion in Section 4.1 and Table 6, the Icelandic year is a leap year, that is a leap week (*sumarauki*) is added, in the following cases [31]:

- The First Day of Summer is on 19 April. Equivalently, the dominical letter is G (or AG).
- The First Day of Summer is on 20 April and the next year is a Gregorian leap year. Equivalently, the dominical letter is A and the next year has dominical letters GF.

This rule is stated in [28] in the equivalent form that there is a *sumarauki* each year that ends on a Monday, or ends on a Sunday and is followed by a leap year. (The conditions are equivalent because 31 December has calendar letter A, cf. Appendix A.1. The use of the last day of the year, instead of for example the first day, avoids problems with the leap day in leap years.)

6.3. **Using concurrents.** By (A.1), we can translate (6.2): If κ is the concurrent a given year, then the First Day of Summer is

$$26-\kappa \quad \text{April}. \tag{6.3}$$

(This also easily follows from (6.1).)

Using (A.3), it follows that the First Day of Summer year y AD is[55]

$$25-\bigl((y+\lfloor y/4\rfloor-\lfloor y/100\rfloor+\lfloor y/400\rfloor+5) \mod 7\bigr) \quad \text{April}. \tag{6.4}$$

6.4. **Using Julian day numbers.** Assume that $\mathbf{JDG}(d,m,y)$ is a function giving the Julian day number (see Appendix A.5) of day d, month m, year y in the Gregorian calendar. (See for example [12] or [16] for the construction of such a function.) By (A.6),

$$\begin{aligned}\mathbf{dowG}(d,m,y)&=(2+\mathbf{JDG}(d,m,y)) \mod 7\\ &=\bigl((1+\mathbf{JDG}(d,m,y)) \mod 7\bigr)+1,\end{aligned} \tag{6.5}$$

and thus, by (6.1), the Julian day number of the First Day of Summer in Gregorian year y can be written

$$\begin{aligned}&\mathbf{JDG}(19,4,y)+\bigl((3-\mathbf{JDG}(19,4,y)) \mod 7\bigr)\\ &=\mathbf{JDG}(25,4,y)-\bigl((3+\mathbf{JDG}(19,4,y)) \mod 7\bigr).\\ &=\left\lfloor\frac{\mathbf{JDG}(22,4,y)}{7}\right\rfloor\cdot 7+3.\end{aligned} \tag{6.6}$$

See further Appendix B.

6.5. **The solar cycle and the Gregorian 400 year cycle.** The solar cycle is disrupted in the Gregorian calendar. In each century, the dominical letters change in the same sequence as for the Julian calendar; but at the turn of the century, three times out of four, a leap day is skipped, which means a jump to a new place in the cycle of dominical letters. It is easily verified that this jump is 16 steps forwards (or, equivalently, 12 steps backwards) plus the usual step forward.

[55] Árnason [4] gives 1739, in words, the equivalent formula

$$18+8-\bigl((y+\lfloor y/4\rfloor-(\lfloor y/100\rfloor-(\lfloor y/400\rfloor+6))) \text{ amod } 7\bigr) \quad \text{April}.$$

year			S1 Apr	S2 May	S3 Jun	S4 Jul	S5 Aug	S6 Sep	W1 Oct	W2 Nov	W3 Dec	W4 Jan	W5 Feb	W6 Mar
	GF		**25**	25	24	28	27	26	**26**	25	25	24	23	25
	E		**24**	24	23	27	26	25	**25**	24	24	23	22	24
	D		**23**	23	22	26	25	24	**24**	23	23	22	21	23
1999	C		**22**	22	21	25	24	23	**23**	22	22	21	20	21
2000	BA		**20**	20	19	23	22	21	**21**	20	20	19	18	20
	G	∗	**19**	19	18	29	28	27	**27**	26	26	25	24	26
	F		**25**	25	24	28	27	26	**26**	25	25	24	23	25
	E		**24**	24	23	27	26	25	**25**	24	24	23	22	23
1700	(D)C		**22**	22	21	25	24	23	**23**	22	22	21	20	22
	B		**21**	21	20	24	23	22	**22**	21	21	20	19	21
	A		**20**	20	19	23	22	21	**21**	20	20	19	18	20
	G	∗	**19**	19	18	29	28	27	**27**	26	26	25	24	25
1800	(F)E		**24**	24	23	27	26	25	**25**	24	24	23	22	24
	D		**23**	23	22	26	25	24	**24**	23	23	22	21	23
	C		**22**	22	21	25	24	23	**23**	22	22	21	20	22
	B		**21**	21	20	24	23	22	**22**	21	21	20	19	20
1900	(A)G	∗	**19**	19	18	29	28	27	**27**	26	26	25	24	26
	F		**25**	25	24	28	27	26	**26**	25	25	24	23	25
	E		**24**	24	23	27	26	25	**25**	24	24	23	22	24
2099	D		**23**	23	22	26	25	24	**24**	23	23	22	21	22
	CB		**21**	21	20	24	23	22	**22**	21	21	20	19	21
	A		**20**	20	19	23	22	21	**21**	20	20	19	18	20
	G	∗	**19**	19	18	29	28	27	**27**	26	26	25	24	26
1799	F		**25**	25	24	28	27	26	**26**	25	25	24	23	24
	ED		**23**	23	22	26	25	24	**24**	23	23	22	21	23
	C		**22**	22	21	25	24	23	**23**	22	22	21	20	22
1898	B		**21**	21	20	24	23	22	**22**	21	21	20	19	21
	A	∗	**20**	20	19	*30*	*29*	*28*	***28***	*27*	*27*	*26*	*25*	26

TABLE 7. Years, dominical letters and beginning of months in the Gregorian version of the Icelandic calendar. The second column shows the dominical letter(s); brackets indicate dominical letters that are missing in 1700, 1800, 1900, 2100, ..., which are not leap years, but present in other years on these lines such as 1728. (These columns are valid only until Dec. 31, and therefore not for the months W4–W6, which are in the next Gregorian year.) A ∗ in the third column marks Icelandic leap years with *sumarauki*. The first days of summer and winter are marked by boldface, and *rímspillir* is marked by italics.

The first two columns in Table 7 show this. For resons of space we have only entered a few years in the first column, but the reader should envision all

intermediate years in each century added in sequence after the first (wrapping around from the last line to the first); moreover, there is a period of 400 years, so 2100 is in the same place as 1700, and so on. For example, since 2000 is on line 5, we have 2001 on line 6, ..., 2099 on line 20 (as shown). Then there is a jump, and 2100 is on line 9 (where 1700 is shown), and the sequence continues from there with 2101 on line 10, and so on. In particular, 2009 is thus on line 14, with dominical letter D and First Day of Summer 23 April. As another example, the years in 1700–2100 that are on the first line are 1720, 1748, 1776, 1816, 1844, 1872, 1912, 1940, 1968, 1996, 2024, 2052, 2080.

The First Day of Summer each year is given by Table 6, and the Icelandic years are thus easily calculated for each line.

There is one exception. The year 1899 (as well as 2299, 2699, ...) has dominical letter A, as given by Table 7 and the discussion above, but the following year (1900) is *not* a Gregorian leap year, and therefore the Icelandic year beginning in 1899 does *not* have a leap week. We thus cannot enter 1899 on the last line (where it would belong, after 1898), and we have instead entered 1898 on the line before. (Year 1899 follows the other years with dominical letter A that are not followed by a Gregorian leap year: lines 5, 11 and 22 in Table 7.)

Rímspillir occurs on the last line of Table 7, that is, as found in Section 6.2 when the next Gregorian year has dominical letters GF. In other words, *rímspillir* ends in Gregorian years with dominical letters GF.

Rímspillir usually occurs with gaps of 28 years (as in the Julian calendar), but because of the 12 step jump backwards in the cycle of dominical letters at the century years that are not leap years, the gap is 40 years for the gaps containing these years. (It is easily checked that the gaps are 40 and not 12, which also would have been conceivable.) It follows that *rímspillir* occurs 13 times in each 400 year period, with intervals of 28 years (10 times) or 40 years (3 times, across the century years that are not Gregorian leap years). The years in 1700–2100 when *rímspillir* begins are 1719, 1747, 1775, 1815, 1843, 1871, 1911, 1939, 1967, 1995, 2023, 2051, 2079.

7. Variations

The preceding sections describe the standard version of the Icelandic calendar, but there have been some variations. The earliest forms of the calendar are discussed in Section 3, and the two different traditions for beginning the winter (on a Friday or a Saturday) are discussed in Section 2.1.

7.1. **Deviations in the printed Icelandic Almanac.** In the printed Icelandic Almanac, which has been published since 1837, the leap week was inserted last in the summer until 1928 [28], [31, p. 346]. (I do not know whether this was based on a tradition existing also before 1837.) This means that the Gregorian dates in Table 1 for S4–S6 were shifted to 22–28 July, 21–27 August and 20–26 September, and the dates given for S4–S6 in Table 7

were shifted a week for the 5 lines with Icelandic leap years. Note that this affects only the last three summer months, which in any case were not much used; the reckoning by weeks was not affected.

The Icelandic Almanac for 1888 forgot to insert the leap week (*sumarauki*); this was corrected the following year [10].

7.2. **A different description.** The Icelandic year is described in this paper as consisting of 12 months with 30 days each, plus 4 extra days (*aukanœtur*), and in leap years also the 7 days *sumarauki*, between the third and fourth summer months; this is in agreement with most descriptions, for example the explanations to the Icelandic Almanac [28] and Schroeter [31].

However, some authors (for example Bilfinger [8], Beckman [1] and Ginzel [17]) give an alternative description where the extra days are included in the third summer month, Sólmánuður, which thus has 34 days in an ordinary year and 41 in a leap year.

I have not found any support for this alternative description in Icelandic sources, neither modern nor medieval. For example, *Íslendingabók* [3] writes about the earliest year (before leap weeks were introduced) "that is 52 weeks, or 12 months, each of 30 nights, and 4 days in addition"[56], and *Bókarbót* [1, p. 78] is even more explicit: it lists all months and the days of week on which they begin: "... And the third month comes on a Monday. Then follow four days which are the days the summer is longer than the winter. And the fourth month comes on a Sunday. ..."[57] Further, this is the natural interpretation of the law in *Grágás* [18] quoted in Section 2.3 (Footnote 18).

In practice, it usually makes no difference whether the last days (*aukanœtur* and, in leap years, *sumarauki*) before Midsummer are regarded as part of month S3 or not. (Especially since months only very rarely have been used for dating.) For example, in the modern Icelandic Almanac [2], only the beginning of each Icelandic month is shown, but the explanations [28] say that *aukanœtur* comes after Sólmánuður.

Appendix A. Some standard calendrical concepts

A.1. **Dominical letters.** This is a device to calculate the day of week of any day in the Julian or Gregorian calendar. (Used since the Middle Ages, and standard in medieval perpetual calendars.) The device consists of two parts, with letters being assigned to both days (without year) and to years.

(i) Each day in a calendar year (except 29 February) is given a *calendar letter* A–G; these letters are assigned in order, with A on 1 January, B on 2 January, ..., G on 7 January, and then repeated throughout the year with A on 8 January, ..., G on 30 December, A on 31 December. Note that this means that in any given year (except leap

[56] þat verþa vicur ii ens setta tegar en monoþr xii. þritøgnattar oc dagar iiii. umbfram [3, Ch. IV]

[57] ... En þriþi manǫðr cemr annan dag viku. Þeim fylgia fiorar nętr þær er sumar er lengra en vetr. En fiorþi manoðr cemr drottins dag. ... [1, p. 78]

years), all Sundays have the same letter, all Mondays the next, and so on. (Here and below we count cyclically, with G followed by A.)

(ii) Every year is given a *dominical letter*, which is the calendar letter assigned to the Sundays that year. A leap year is given two dominical letters; the first valid in January and February and the second in March–December. (The second letter is always the one coming before the first.)

Since 365 days is one day more than 52 weeks, the dominical letter of the following year is the letter before the present one.

For example, 2009 begins on a Thursday. Hence, A=Thursday, B=Friday, C=Saturday, D=Sunday, E=Monday, F=Tuesday, G=Wednesday, and the dominical letter of 2009 is D. (This is given in many almanacs for 2009, for example [2].) The dominical letters of 2008 (a leap year) are FE, and 2010 has C.

The traditional method to calculate the dominical letter for a given year is to use the solar cycle, Section A.3. An alternative, suited for computer calculations, is to use an algorithm to calculate the day of week of 1 January the given year, from which the dominical letter is easily found. See for example [27] for detailed algorithms. (Or use (A.1)–(A.3) below.)

The use of the letters A, ..., G as symbols is traditional but of course arbitrary. Numbers (as in our calculations above) have been used occasionally, and the first seven runes have sometimes been used in other parts of Scandinavia, in particular on the Swedish rune staffs, see [24].

A.2. **Concurrent.** An alternative to the dominical letter is the *concurrent*, which for each year is a number 1–7. (The concurrent was also a standard tool in medieval time-reckoning.) The concurrent of a year signifies the day of week of 24 March, with 1 = Sunday, 2 = Monday, ..., 7 = Saturday. Since 24 March has dominical letter F, there is a one-to-one correspondence between dominical letters and concurrents, with dominical letter F corrsponding to concurrent 1, and so on. (For leap years, it is the second dominical letter that is used.) In formulas, it is easily seen that if we denote the concurrent by κ, and let δ be the number corresponding to the dominical letter of the year (the second one for a leap year), then the sum $\kappa+\delta$ equals 7 or 14, and is thus divisible by 7; this can be written

$$\kappa+\delta\equiv 0 \pmod 7. \tag{A.1}$$

The concurrent κ of year y AD in the Julian calendar is given by

$$\kappa = (y+\lfloor y/4\rfloor+4) \mod 7. \tag{A.2}$$

In the Gregorian calendar, the corresponding formula is

$$\kappa = (y+\lfloor y/4\rfloor-\lfloor y/100\rfloor+\lfloor y/400\rfloor+6) \mod 7. \tag{A.3}$$

A.3. **Solar cycle.** The Julian calendar has a well-known cycle of 28 years for the days of week (and thus of the dominical letters), known as the *solar cycle*. The years in the cycle are numbered 1–28, and year y AD has number $(y + 9) \mod 28$ in the cycle. The first year in each cycle is a leap year, with 1 January on a Monday, so its dominical letters are GF. Hence year 2 in each cycle begins on a Tuesday and has dominical letter E, and so on. The full cycle of dominical letters is given in Table 5.

Year 1 AD is number 10 in the solar cycle. Thus (by coincidence) 1000 AD is number 1. Hence, the number in the solar cycle of year y AD is

$$(y + 9) \mod 28 = \big((y - 1000) \mod 28\big) + 1. \tag{A.4}$$

The solar cycle was disrupted by the Gregorian calendar reform. Although the number in the solar cycle still is calculated by the same rule, and given in many almanacs (for example [2]), the relation between the number in the solar cycle and the dominical letters changes from century to century, and the solar cycle has little practical use in the Gregorian calendar.

A.4. **Lunar cycle.** The *lunar cycle* is a 19 year cycle for the phases of the moon. This is in reality approximate only, but is treated as an exact cycle in the ecclesiastical calculation of Easter in the Julian calendar, see for example [27, Chapter 29]. (The Gregorian calendar uses the same basic cycle, but introduces certain corrections.) The number of a year in the lunar cycle is called the *golden number*, usually written with Roman numerals (in the range I–XIX). Year 1 AD has golden number II; thus year y AD has golden number

$$(y + 1) \mod 19. \tag{A.5}$$

The lunar cycle is not relevant for the Icelandic calendar as such, but it is important for the date of Easter, see Appendix C.1.

A.5. **Julian day number.** The Julian day number (which we abbreviate by JD) is a continuous count of days, beginning with JD 0 on 1 January 4713 BC (Julian).[58] Such a numbering is very convenient for many purposes, including conversions between calendars where it often is convenient to calculate the Julian day number as an intermediary result. The choice of epoch for the day numbers is arbitrary and for most purposes unimportant; the conventional date 1 January 4713 BC Julian (-4712 with astronomical numbering of years) was originally chosen by Scalinger in 1583 as the origin of the Julian

[58]Astronomers use a slightly different version [16, Section 12.7]. Their Julian day numbers change at noon UT (GMT); moreover, they add a fractional part to show the exact time; thus obtaining the *Julian date*, which is a real number that defines the time of a particular instance.

In calendrical calculations, however, the Julian day number is assigned to calendar days, regardless of when the days begin and end in that calendar, and what the time then is at Greenwich.

period, a (cyclic) numbering of years, and was later adapted into a numbering of days. See further [16, Section 12.7].[59]

For example, 28 November 1700, the first day the Gregorian calendar was used in Iceland, has JD 2342304, and the day before (16 November 1700 Julian) JD 2342303; the Icelandic First Day of Summer 2009, 23 April 2009, has JD 2454945.

Since JD 0 was a Monday, that is day of week number 2 in our numbering, the day with Julian day number JD has day of week number

$$(2+\mathrm{JD}) \mod 7 = \bigl((1+\mathrm{JD}) \mod 7\bigr) + 1. \tag{A.6}$$

Appendix B. Some formulas for calendar conversions

To convert Icelandic dates to or from dates in any other calendar, it is as said above convenient to first convert to Julian day numbers. We give some mathematical formulas for conversions between the Icelandic calendar and Julian day number. We regard, as in the rest of this paper, the Icelandic year as starting with First Day of Summer; hence Harpa is month 1. Icelandic years are usually not numbered, but for the formulas we number the year by the standard numbering AD of the Julian or Gregorian year where the First Day of Summer falls (and with it the larger part of the Icelandic year).

It is straightforward in principle to calculate the Julian day number of a given day in a given month and year in the Julian or Gregorian calendar, but the formulas are a bit complicated because of the somewhat irregular lengths of the months and the varying length of February; see for example [12, Chapters 2 and 3] or [16] for explicit formulas. In the calculations in Sections 5 and 6, we only need the Julian day number for days in April, and for this case we have simple formulas: the Julian day number of day d, month 4 (April), year y, is for the Julian calendar

$$\mathbf{JDJ}(d,4,y) = 1721148 + d + 365\cdot y + \lfloor y/4\rfloor, \tag{B.1}$$

and for the Gregorian calendar

$$\mathbf{JDG}(d,4,y) = 1721150 + d + 365\cdot y + \lfloor y/4\rfloor - \lfloor y/100\rfloor + \lfloor y/400\rfloor. \tag{B.2}$$

Using (5.6) and (6.6), we thus find formulas for the Julian day number of the First Day of Summer in (the Julian or Gregorian) year y, in the Julian

[59] Dershowitz and Reingold [12] use another day number, denoted by RD, with another epoch: RD 1 is 1 January 1 AD (Gregorian) which is JD 1721426. Consequently, the two day numbers are related by JD = RD + 1721425.

or Gregorian version of the Icelandic calendar:

$$\mathbf{FDSJ}(y) = \left\lfloor \frac{1721160 + 365 \cdot y + \lfloor y/4 \rfloor}{7} \right\rfloor \cdot 7 + 3 \tag{B.3}$$

$$= \left\lfloor \frac{1461 \cdot y}{28} \right\rfloor \cdot 7 + 1721163, \tag{B.4}$$

$$\mathbf{FDSG}(y) = \left\lfloor \frac{1721172 + 365 \cdot y + \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor}{7} \right\rfloor \cdot 7 + 3 \tag{B.5}$$

Let **FDS** denote the appropriate one of these (**FDSJ** if $y < 1700$, otherwise **FDSG**). Using this function we can then calculate the Julian day number of day d, month m, year y (with the conventions above) as

$$\mathrm{JD} = \begin{cases} \mathbf{FDS}(y) + d - 1 + 30 \cdot (m-1) & \text{if } m \le 3, \\ \mathbf{FDS}(y+1) + d - 1 - 30 \cdot (13-m) & \text{if } m \ge 4. \end{cases} \tag{B.6}$$

(Thus counting backwards from the next First Day of Summer for $m \ge 4$.) This applies also to *aukanætur* and *sumarauki* if we, artificially, regard them as days 31–34 and 35–41 of month 3 (see Section 7.2).

In particular, the First Day of Winter is given by

$$\mathbf{FDW}(y) = \mathbf{FDS}(y+1) - 180. \tag{B.7}$$

As another example, *bóndadagur* (the first day of Þorri, *miður vetur*) in the Julian or Gregorian year y has Julian day number $\mathbf{FDS}(y) - 90$. (Note that this is in January, and thus in Icelandic year $y-1$ with our numbering.)

For the more common reckoning with weeks, we number as above the days of week with 1 = Sunday. Then day of week d in week w in the summer or winter of year y has Julian day number

$$\mathrm{JD} = \begin{cases} \mathbf{FDS}(y) + 7 \cdot (w-1) + \big((d+2) \mod 7\big) & \text{if summer week}, \\ \mathbf{FDW}(y) + 7 \cdot (w-1) + \big(d \mod 7\big) & \text{if winter week}. \end{cases} \tag{B.8}$$

Conversely, to convert a Julian day number JD to an Icelandic date, we first determine the year y. There is no simple formula, so we first find an approximate year y_0, for example by

$$y_0 = \left\lfloor \frac{\mathrm{JD} - 1721000}{365.25} \right\rfloor. \tag{B.9}$$

This will (for the next 20 000 years) give either the correct year y or $y+1$, so we calculate $\mathbf{FDS}(y_0)$; if $\mathbf{FDS}(y_0) \le \mathrm{JD}$, we have $y = y_0$, otherwise $y = y_0 - 1$.

To find the number w of the week, we first calculate $\mathbf{FDW}(y)$ by (B.7); if $\mathrm{JD} < \mathbf{FDW}(y)$ we are in the summer, and otherwise in the winter *misseri*. Then

$$w = \begin{cases} \lfloor (\mathrm{JD} - \mathbf{FDS}(y))/7 \rfloor + 1 & \text{if summer}, \\ \lfloor (\mathrm{JD} - \mathbf{FDW}(y))/7 \rfloor + 1 & \text{if winter}. \end{cases} \tag{B.10}$$

To find the month m, we compare with Midsummer ($\mathbf{FDW}(y)-90$):

$$m=\begin{cases}\lfloor(\mathrm{JD}-\mathbf{FDS}(y))/30\rfloor+1 & \text{if } \mathrm{JD}<\mathbf{FDS}(y)+90,\\ 3\ (\textit{aukanætur}) & \text{if } \mathbf{FDS}(y)+90\le \mathrm{JD}<\mathbf{FDS}(y)+94,\\ 3\ (\textit{sumarauki}) & \text{if } \mathbf{FDS}(y)+94\le \mathrm{JD}<\mathbf{FDW}(y)-90,\\ \lfloor(\mathrm{JD}-\mathbf{FDW}(y)+210)/30\rfloor & \text{if } \mathrm{JD}\ge\mathbf{FDW}(y)-90.\end{cases} \tag{B.11}$$

The day d of the month is $1+$ the remainder in the division; alternatively, it is $\mathrm{JD}+1-\mathrm{JD}_1$, where JD_1 is the Julian day number of the first day in the month, calculated as above. (We again for mathematical convenience treat *aukanætur* and *sumarauki* as parts of month 3; they can easily be treated separately.)

Appendix C. Easter

C.1. **Position of Easter in the Julian version (12th c. – 1700).** In the Julian (and Gregorian) calendar, Easter Day is a Sunday in 22 March – 25 April. This is a range of 35 days, that is, exactly 5 weeks.

To translate the dates to the Icelandic calendar, it is convenient to first consider a day with the same day of week as the First Day of Summer. We choose Maundy Thursday, which is 3 days before Easter Day; thus a Thursday in 19 March – 22 April. Again, this is a range of 35 days, which can be divided into 5 possible weeks for Maundy Thursday: 1: 19–25 March; 2: 26 March – 1 April; 3: 2–8 April; 4: 9–15 April; 5: 16–22 April. Note that the fourth possible week is exactly the range 9–15 April containing the First Day of Summer in the Julian version. Hence, if Easter Day is in the 4th possible week, then the First Day of Summer coincides with Maundy Thursday. In this case, Easter Day is 3 days after the First Day of Summer (that is, 4 Harpa). For the four other weeks, we find that the First Day of Summer is exactly 3, 2 or 1 week(s) after, or 1 week before Maundy Thursday. Consequently, the five possibilities for the First Day of Summer are, as stated in *Rím II* [1, p. 170 §§163–164], the 3rd week after Easter, the 2nd week after Easter, the 1st week after Easter, Maundy Thursday, the week before Palm Sunday.

Conversely, this yields the 5 possibilities in Table 8 for the position of Easter in the Icelandic year. We can also say that Easter Day is the 3rd last Sunday in the winter, the second last, the last, the first Sunday in Summer, and the second Sunday in Summer, respectively.

All other holidays (and other special days) that are governed by Easter can now be placed in the Icelandic calendar by counting backwards or forwards from Easter Day. Note that this yields only 5 possibilities for each day, as a consequence of the fact that each day in the Icelandic calendar has a fixed day of week. (And neither Julian leap days, Icelandic leap weeks nor *rímspillir* affects the calculations.)

	Maundy Thursday	Easter Day
1	3 weeks before FDS	24th week of winter (13 Einm.)
2	2 weeks before FDS	25th week of winter (20 Einm.)
3	1 week before FDS	26th week of winter (27 Einm.)
4	FDS	1st week of summer (4 Harpa)
5	1 week after FDS	2nd week of summer (11 Harpa)

TABLE 8. The five possible positions of Easter in the Icelandic calendar (Julian version). (FDS = First Day of Summer.)

These five possibilities are discussed in *Rím I* [1, pp. 25 §30]; the discussion there centers on Shrove Sunday (*Quinquagesima*) at the beginning of Lent (7 weeks before Easter Day, and 3 days before the Lent starts on Ash Wednesday), and the possibilities are called first – fifth *fǫstugangr* ("Lent entrance"). We therefore give a table for Shrove Sunday too, which is easily derived from Table 8; cf. the similar table by Beckman [1, Tab. VII] and [5, Tab. V]. (When considering Shrove Sunday, as in *Rím I*, it is convenient that Gói begins on a Sunday.) We also give the Old Icelandic expressions from [1, *Rím I* p. 25 §30] and the Easter table [1, pp. 69–70] for the 5 weeks. *Rím II* [1, pp. 139–140 §118] gives the Latin abbreviations QÞ, PG, SG, TG, QG (*quarta dominica Þorra, prima dominica Gói, secunda dominica Gói, tertia dominica Gói, quarta dominica Gói*).

	Shrove Sunday	Old Icelandic [1, *Rím I*]	
1	last Sunday in Þorri (24 Þorri)	*vika lifer þorra*	one week left of Þorri
2	first Sunday in Gói (1 Gói)	*i aundverda goe*	beginning of Gói
3	second Sunday in Gói (8 Gói)	*vika af goe*	one week of Gói
4	third Sunday in Gói (15 Gói)	*i midia goe*	middle of Gói
5	fourth Sunday in Gói (22 Gói)	*vika lifer goe*	one week left of Gói

TABLE 9. The five possible positions of Shrove Sunday in the Icelandic calendar (Julian version).

Rím I summarizes: "There are five *fǫstugangar* according to week reckoning, but 35 according to day reckoning, but the people use only the week reckoning."[60].[61]

[60]Fimm ero fostu gangar ath vikna tale, enn halfur fiorde togr ath dagha tali, enn vikna tal eitt þarf til alþydu tals. [1, p. 25, §30]

[61]While Easter Day can fall on 35 days in the Julian calendar, Shrove Sunday can actually fall on 36 days: 1–29 February and 1–7 March; this further variation is caused by the leap day that some years is inserted between Shrove Sunday and Easter Day. Thus the range is more than 5 weeks, and there is no corresponding set of 5 *fǫstugangar* in the Julian calendar. (The same applies to for example Ash Wednesday and Septuagesima.) Beckman [1, p. CXC] discusses this and explains the simpler situation in the Icelandic

Rím I [1, pp. 25–28 §§31–33] and *Rím II* [1, pp. 163–166 §§151–154] give the *fǫstugangr* for each year in the 19-year lunar cycle (that is, for each golden number); usually there are two possibilities depending on the dominical letter of the year, and this is completely described. For example, for the first year in the lunar cycle (golden number I, see Appendix A.4), the ecclesiastical calendar reckons a new moon 23 March, and thus a full moon 5 April (always 13 days later). Easter Day is the next Sunday, which thus is 6–12 April. Of these dates, 6–11 April are in the 3rd *fǫstugangr*, and 12 April in the 4th. Since 12 April has calendar letter D, the result (stated in *Rím I* [1]) is that the *fǫstugangr* is "*vika er af goe*", unless the dominical letter is D when it is "*i midia goe*", cf. Table 9. Repeating this argument for each golden number yields the result in Table 10, where the *fǫstugangr* for a year is found by looking at the row for the golden number and finding the column containing the dominical letter (the second dominical letter for a leap year) if it exists, and otherwise the column with X. This table is in accordance with the lists in *Rím I* [1, pp. 25–28 §§31–33] and *Rím II* [1, pp. 163–166 §§151–154], although we for convenience have added Julian dates for new and full moon and for Easter Day.

The Icelandic calendar seems well adapted to the ecclesiastical calendar and the varying position of Easter. The fact that Easter Day (and other days depending on it) can fall on only 5 different days in the Icelandic calendar (and not 6) is, as follows from the discussion above, due to two facts, apart from the central property that each day has a fixed day of week:

(1) The First Day of Summer is constrained to a period of 7 days. (An alternative would have been to constrain the First Day of Winter to 7 days in the Julian calendar; this would have given *rímspillir* in months W6–S3 instead, and a range of 8 days for the First Day of Summer. Cf. Beckman [5, p. 29], where this is discussed in terms of choosing between the years 8 and 9 in the solar cycle for *sumarauki*, see Table 5.)
(2) The 7 day period for First Day of Summer is one of the 5 weeks that the range for Maundy Thursday can be divided into (or shifted from them by a number of whole weeks). Again, see Beckman [5, pp. 30–31], where this is discussed in terms of choosing the dominical letter associated to *sumarauki*.

Beckman [1, p. CXC] and [5, pp. 30–31] believes that this best possible adaption of the Icelandic calendar to the position of Easter is not a coincidence, and that the Icelandic calendar was tied to the Julian calendar in a skillfully chosen way in order to achieve this system of 5 *fǫstugangar*.

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calendar by saying that the exceptions caused by the leap day in the Julian calendar are cancelled by the exceptions caused by *rímspillir*. While this is correct, is is simpler to observe that the Julian leap days does not appear in Icelandic calendar, and (as above) count backwards from Easter Day in the Icelandic calendar instead of doing it in the Julian calendar and then converting the dates to the Icelandic calendar as Beckman [1] does.

	new moon	full moon	Easter Day	1 *lif. Þ*	2 *au. G*	3 *vik. G*	4 *mid. G*	5 *lif. G*
I	23/3	5/4	6–12/4			X	D	
II	12/3	25/3	26/3–1/4	ABC	X			
III	31/3	13/4	14–20/4				X	DE
IV	20/3	2/4	3–9/4		BC	X		
V	9/3	22/3	23–29/3	X	D			
VI	28/3	10/4	11–17/4			C	X	
VII	17/3	30/3	31/3–6/4		X	DE		
VIII	5/4	18/4	19–25/4					X
IX	25/3	7/4	8–14/4			X	DEF	
X	14/3	27/3	28/3–3/4	C	X			
XI	2/4	15/4	16–22/4				ABC	X
XII	22/3	4/4	5–11/4			X		
XIII	11/3	24/3	25–31/3	X	DEF			
XIV	30/3	12/4	13–19/4				X	D
XV	19/3	1/4	2–8/4		ABC	X		
XVI	8/3	21/3	22–28/3	X				
XVII	27/3	9/4	10–16/4			BC	X	
XVIII	16/3	29/3	30/3–5/4		X	D		
XIX	4/4	17/4	18–24/4				C	X

TABLE 10. The *fǫstugangar* according to the golden number (lunar cycle) and dominical letter. X means all other dominical letters. (Julian version.)

C.2. **Position of Easter in the Gregorian version (1700 – present).** The Icelandic Almanac [2] has a section on the position of Easter among the Icelandic months. Usually, Easter is in Einmánuður, but occasionally Easter is in Góa or in Harpa (that is, after the First Day of Summer and in the summer *misseri*). This is called *góupáskar* and *sumarpáskar*, respectively. In the Gregorian calendar, Easter Day is always a Sunday in the range 22 March – 25 April. It is seen from Table 1 that *góupáskar* occurs when Easter Day is on March 22, 23 or 24 (then Einmánuður begins 2 days later, so Easter Day is on 29 Góa), and that *sumarpáskar* occurs when Easter Day is on April 22, 23, 24 or 25 (then Harpa begins 3 days earlier, so the First Day of Summer coincides with Maundy Thursday, and Easter Day is on 4 Harpa).

As said in [2], *góupáskar* occurs on the average only about 1 year in 35, but rather irregularly with gaps ranging from 11 to 152 years; the last time was 2008 and the next is 2160. Similarly, *sumarpáskar* occurs on the average almost once in 15 years, with gaps raging from 3 to 41 years; the last time was 2000 and the next is 2011.

Appendix D. Some rules from *Rím I* and *Rím II*

Rím I and *Rím II* give several rules on the position in the Icelandic calendar (Julian version) of holidays and other days in the ecclesiastical calendar. We quote and comment some of them here, as examples both of the calendar and of how it was described. (Most rules are given in both *Rím I* and *Rím II*, and sometimes more than once in *Rím II*, in identical or similar formulations. We do not give references to all places.)

Christmas. "Christmas Day shall be in the 11th week of winter, whatever day of week it is, except when it is a Friday or a Saturday, then it is in the 10th week, and so also when it is *rímspillir*."[62]

Counting backwards 10 weeks (70 days) from Christmas Day (25 December), we see that this is equivalent to saying that 16 October is in the first week of winter, unless it is a Friday or Saturday or it is *rímspillir*. This is mainly correct; by Table 1, winter begins a Saturday 11–18 October, so 16 October is in the first week unless winter begins 17 October (then 16 October is a Friday) or 18 October (*rímspillir*). However, the quoted rule also makes an exception when Christmas Day is on a Saturday. This can be explained by assuming that the weeks here are reckoned as starting on Sundays.

This rule has been debated by Björnsson [10] and Beckman [1]; Björnsson [10, p. 290] interprets it differently, partly based on a different formulation in some manuscripts, but Beckman's [1, p. LXXII] refutation of this seems well founded.

Rím II has an essentially identical formulation [1, p. 85 §7], but also a version without the exception for Saturday: "Christmas Day is always in the 11th week of winter, except when it is on a Friday, then it is in the 10th week, or on a Thursday if a leap year comes after"[63] (the last case is *rímspillir*). This version is correct if the weeks are reckoned as starting on Saturdays (as winter weeks usually are).

End of Christmas. "There should always be one Wednesday between Christmas and Midwinter, except in *rímspillir*, then there are two Wednesdays."[64]

The end of Christmas is Epiphany, 6 January. Since Midwinter (1 Þorri) is on a Friday, this rule means that it should be on the first Friday after 8 January (that is, 9–15 January), except in *rímspillir*, which is in agreement with Table 1.

This rule is also stated several times in *Rím II* [1, p. 85 §7; p. 129 §88; p. 156 §137; p. 169 §162], with minor variations such as: "There should always

[62] Iola dagur skal vera i enne XI viko vetrar, hvernge dagh sem hann er i viko, nema hann se fostu dagh eda þvat dagh, þa er hann i tiundu viko, ok sva þa er rimspiller er. [1, *Rím I* p. 24 §29]

[63] er iola dagur æa elliptu viku vetrar iafnan, nema hann se æa faustu dag, þa er hann æa tiundu viku vetrar eda fimtu dag, ef hlaupar ferr epter. [1, p. 129 §88]

[64] Einn skal midviku dagur avallt æa milli iola ok mids vetrar, nema i rimspille, þa verda II midviku dagar. [1, *Rím I* p. 24 §29]

be one Wednesday between Epiphany and Þorri, except when Epiphany is on a Tuesday and a leap year follows, then there should be two."[65]

A related rule is in *Rím II*: "That Friday [Midwinter] is the second from the 8th day of Christmas [1 January], except in the 9th year of the solar cycle, then it is the third."[66] Beckman [1, p. 84 n. 7] comments that "9th year" is correct (although some manuscripts have "8th"): *rímspillir* begins in the 8th year, but a new Julian year just started on 1 January.

Candlemas. "Candlemas is in the 4th week of Þorri if it is on a Sunday or Monday, and it is not *rímspillir*, but in the 3rd week of Þorri otherwise"[67]

Since Þorri begins a Friday 9–16 January, Candlemas (2 February) is 18–25 Þorri (18 Þorri only at *rímspillir*); of these dates, only 24 Þorri is a Sunday and 18 and 25 Þorri are Mondays. Hence it seems that this rule reckons the "4th week of Þorri" as beginning on Sunday 24 Þorri, so it seems that we here have an instance of reckoning weeks beginning on Sundays.

First Day of Winter. "Winter comes on the Saturday that is next before St Luke [18 October], but on it if a leap year comes after."[68]

The exception evidently applies only when St Luke is on a Saturday; if this happens and the next year is a (Julian) leap year, then we have *rímspillir*. (Otherwise, if winter begins on 11 October, the next summer would start too early on 8 April. Alternatively, St Luke is on a Saturday when the dominical letter of the year is E, which together with the next year being a leap year is the condition for *rímspillir*, see Section 5.2.) Hence this rule is in agreement with Table 1.

First Day of Summer. "Summer must not come earlier than before Palm Sunday, and not later than in the second week after the Easter week."[69]

Easter week is the week beginning on Easter Day, so this gives the range from the Thursday before Palm Sunday, that is in the second week before Easter, to the third week after Easter Day, in agreement with Appendix C.1. (Since Easter week begins on a Sunday, the weeks here are obviously reckoned from Sunday.)

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[65]Einn midvikudagr skal verda milli þrettanda dags iola ok þorra iafnan, nema hinn þrettandi dagr se æa þridiu degi viku ok komi hlaupar eptir um vorit, þa skulu tveir. [1, p. 156 §137]

[66]Sæa fria dagur er anar fra 8. degi iola, nema hin 9. vetur solar alldar, þa verdur sa hin 3. [1, p. 84 §6]

[67]A fiordu viku þorra er kyndil messa, ef hun er drottins dagh eda annann dagh viku, svo ath eighe se rimspiller, en æa þridiu viku þorra hvernge dagh annarra [1, *Rím I* p. 24 §29]

[68]Vetur kemur laugar dag, er næstur er fyri Lukas messo, en hana sialfa, ef hlaupar ferr epter. [1, *Rím II* pp. 128–129 §87]

[69]Sumar ma eigi koma fyrr enn fyri palma dagh, ok eigi sidar enn a annarre viko efter pascha viko. [1, *Rím I* p. 22 §26]

Ember Days. "The Ember days in the autumn shall always be held in the fifth week, whatever day of week Exaltation of the Cross [14 September] is, except if it is on a Saturday or Sunday, then the Ember Days shall be held in the fourth week. But if that happens, that Exaltation of the Cross is on a Sunday, and also one should add [a week] to that summer, then the Ember Days are in the fifth week, and not in the fourth, and that winter is called *rímspillir*."[70]

Since these Ember Days are close to the Exaltation of the Cross in the middle of September, the weeks here are evidently reckoned backwards from the end of summer; thus the fifth week comes before the fourth. The Ember Days are always a Wednesday, Friday and Saturday; they thus begin on the Wednesday in the fifth or fourth week before the First Day of Winter, which means 31 or 24 days before the First Day of Winter (which is a Saturday), and thus 1 day before or 6 days after the beginning of the last summer month. (This holds for any reasonable reckoning of the weeks.) To translate the rule to the Julian calendar, we refer to Table 1. Since the last summer month S6 begins on a Thursday 11–18 September (with 18 September in *rímspillir*), it is easily checked that 14 September is a Saturday or Sunday if and only if the month begins on 11, 12 or 18 September, and that it begins on 18 September if and only if 14 September is on a Sunday and there has been a leap week in the summer, that is, the exception given in the rule. It follows that the rule gives the Wednesday 12–18 September; see Beckman [1, Tab. III p. CLXXXVI]. (Beckman [1, p. 23 n. 2] remarks that this differs slightly from the standard rule, which is the first Wednesday after 14 September, but that it seems to reflect actual Icelandic usage during the 12th century.)

Rímspillir. "*Rímspillir* is that at Althingi, St John Baptist [24 June] is on a Tuesday and a leap year comes in the spring after, and then [a week] shall be added in the summer. Then Exaltation of the Cross [14 September] is on a Sunday in the 5th week before winter. Winter comes on St Luke [18 October] and that is a Saturday. Christmas Day [25 December] is a Thursday in the 10th week. Midwinter comes three days after the Octave of Epiphany [13 January], and is a Friday first in Þorri. Candlemas [2 February] is a Monday in the third week."[71]

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[70] Imbro dagha um haust skal hallda avallt æa fimto viko, hvernge dagh sem crucis messa verdr i viko, nema hun se þvatt dagh eda drottins dagh, þa skal hallda imbro dagha æa fiordu viko. Enn ef þat berr saman, ath cross messa er drottins dagh, enda skule þa vid sumar leggia þat sumar, þa ero ymbro dagarner æa fimtu viku, enn eighi æa fiordu, ok heiter sa vetur rimspiller. [1, *Rím I* p. 23 §27]

[71] rimspiller er, ath um þingh skal Ions messa vera þridia dagh viko, ok skal vid sumar leggia, þviat hlaup ar kemur efter um vorit. Þa er crucis messa um haust drottins dagh æa fimtu viku fyrer vetur, enn vetur kemr Lucas messo, ok er hun þvott dagh. Iola dagur er fimta dagh viku i tiundo viko. Midr vetur kemr efter enn atta dagh fra hinum þrettanda þrimur nottum, ok er faustu dagur fyrstur i þorra. Er kyndil messa annan dag viko i þridiu viko [1, *Rím I* pp. 23–24 §§27–28]

This is all easily verified. In the last sentence, "third week" evidently means in Þorri, or equivalently, after Midwinter.

Appendix E. Special days

We give a list of some days, or short periods of several days, with special names in the Icelandic calendar. (Some of these are already mentioned above. Days determined by the Julian or Gregorian calendar are not included.) Almost all are described in the explanation to the Icelandic Almanac [28] (where sometimes further information is given), and many are given in the almanac [2]; some exist already in *Rím I* [1]. Traditions connected to some of them are described by Björnsson [9]. For convenience, for most of the days we also give (in parentheses) the date by Icelandic month and day (for example 1 Harpa), although this form of dating has never been used in Iceland, see Section 2.3.

***aukanœtur*:** 4 extra days inserted after (or at the end of, see Section 7.2) the third summer month Sólmánuður. They thus begin on the Wednesday in the 13th week of summer (90 days after the First Day of Summer). In the Gregorian version, the beginning is on the Wednesday in 18–24 July.

bóndadagur (Husband's day): The first day of Þorri. (1 Þorri.) (The same as *miður vetur*.) In the Gregorian version, a Friday in 19–26 January.

fardagar (Flitting Days): The first 4 days (Thursday–Sunday) of the 7th week of summer [1, *Rím I* pp. 22–23 §26]. (13–16 Skerpla.) These were the days when tenant farmers could move from one farm to another [9, p. 29]. In the Julian version, the first day is the Thursday in 21–27 May. In the Gregorian version, the first day is the Thursday in 31 May – 6 June.

fyrsti vetrardagur (First Day of Winter): The first day in the winter *misseri*. Equivalently, the first day in the first winter month Gormánuður. (1 Gormánuður.) In the Gregorian version, a Saturday in 21–28 October. (Another tradition begins winter on a Friday, see Section 2.1).

***góuþrœll*:** The last day of Góa. (30 Góa.) In the Gregorian version, a Monday in 19–25 March.

konudagur (Wife's Day): The first day of Góa. (1 Góa.) In the Gregorian version, a Sunday in 18–25 February.

***miðgóa*:** Third Sunday in Góa. (15 Góa.)

miðsumar (Midsummer): First day in the 4th summer month, Heyannir. (1 Heyannir.) (In the Middle Ages, perhaps also a name for this month, or for the beginning of it, see Section 2.3.) Equivalently, Sunday in the 14th week of summer, except in (Icelandic) leap years,

when *sumarauki* is inserted just before *miðsumar*, which then is Sunday in the 15th week of Summer. In the Gregorian version, a Sunday in 23–30 July.

miður vetur (Midwinter): First day in the 4th winter month, Þorri. (1 Þorri.) Equivalently, Friday in the 13th week of winter. (That is, 90 days after the First Day of Winter.) In the Julian version, a Friday in 9–16 January. In the Gregorian version, a Friday in 19–26 January.

***miðþorri*:** Third Friday in Þorri. (15 Þorri.)

sumarauki (leap week): A leap week inserted after *aukanœtur* just before *miðsumar*.[72] In the Gregorian version, it begins on 22 July, when that day is a Sunday, or on 23 July, when that day is a Sunday and the next Gregorian year is a leap year.

sumardagurinn fyrsti (First Day of Summer): The first day in the summer *misseri*. Equivalently, the first day in the first summer month Harpa. (1 Harpa.) In the Gregorian version, the Thursday in 19–25 April. (A public holiday in Iceland.)

***sumarmál*:** The last 5 days (Saturday–Wednesday) of the winter *misseri*, just before the First Day of Summer. Equivalently, the incomplete 26th week of winter. (26–30 Einmánuður.) In the Gregorian version, *sumarmál* begins on the Saturday 14–20 April. (Earlier used for the beginning of summer; no precise definition is known [22, Første vinterdag, sommerdag], perhaps at least sometimes the first day [21] or the first four days [39, p. 59].)

***vápnatak*:** In the Middle Ages, the Thursday the Althingi ended, that is, two weeks after the beginning of *þingvikur* [31, p. 329].

vetrarkoma (Winter beginning): The First Day of Winter. (The same as *fyrsti vetrardagur*). (1 Gormánuður.)

veturnœtur (Winter Nights): The last two days (Thursday and Friday) of the summer *misseri*, just before the First Day of Winter. (29–30 Haustmánuður.) (Also used less specifically for the period around the beginning of winter.) Since the summer *misseri* is 26 weeks + 2 days in an ordinary year, and 27 weeks + 2 days in a leap year, this could be regarded as the last, incomplete week of the summer *misseri*. In the Gregorian version, *veturnœtur* fall in 19–27 October.

þingvikur (Thing Weeks): In the Middle Ages, the dates for the Althingi (which lasted two weeks). Until 999 (or perhaps 998 [13]), the Althingi began with the 10th week of summer (that is, it started on the Thursday 9 weeks after the First Day of Summer), but was then moved a week to the 11th week (starting 10 weeks after the First Day of Summer) [3, Ch. VII], [1, p. 23 §26], [5, p. 26]; when the

[72] This is the original position, and is the position today, but in Icelandic almanacs until 1928, it was inserted just before the First Day of Winter instead, see Section 7.1.

calendar had become fixed to the Julian, this was the Thursday in 18–24 June. In 1262, when Iceland became a Norwegian dependency, the day was changed to the day before SS Peter and Paul (that is, 28 June in the Julian Calendar), and the connection with the Icelandic calendar was broken [1, *Rím II* p. 84 §4], [31, p. 322 n. 2].

There were also other, regional, things. [1, *Rím II* p. 84 §4] says that *vorþing* ("Spring thing") begins 5 weeks and two days after the First Day of Summer. (8 Skerpla.) In the Julian version, this is the Saturday in 16–22 May. *Grágás* [18, §56 p. 96] is less specific and allows the interval 4–6th week of summer for the *vorþing*.

***þorraþrœll*:** The last day of Þorri. (30 Þorri.) In the Gregorian version, a Saturday in 17–24 February.

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