Schubert filtration for simple quotients of generalized Verma modules

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1991 Math. Subj. Classification: primary 17B35, secondary 22E47

1 Introduction

Schubert filtration and Demazure character formula form a classical part in the theory of finite-dimensional representations of complex semi-simple finite-dimensional Lie algebras (and, more generally, simple algebraic groups, see [A, J]). Originally, this formula appears in [De] as an improvement of the Weyl character formula describing the character of a simple finite-dimensional module. Schubert filtration arises in a canonical way together with the Demazure formula.

Recently, an analogue of the Weyl character formula for the character of a simple quotient of the so-called α-stratified generalized Verma module (GVM) over a simply-laced Lie algebra was obtained ([FM1]). The obtained formula differs from the Weyl formula by the factor corresponding to a subalgebra of $U(\mathfrak{N}_-)$ acting torsion-free on the simple module. This result stimulated an investigation to find an analogue for the Demazure formula and to construct the corresponding filtration of the simple module.

In the present paper we present an analogue of the Demazure formula and the corresponding filtration (Schubert filtration) for a simple quotient of an α-stratified GVM over a simple simply-laced finite-dimensional complex Lie algebra. The proof of the formula is relatively easy and based on the classical Demazure formula and the character formula obtained in [FM1]. However, our construction of the corresponding filtration is not trivial and crucially uses Mathieu's localization of $U(\mathfrak{G})$ ([Ma]), which allows us to reduce our problem (in part) to classical Verma modules. Unlike the classical case, a simple α-stratified module does not contain clear analogues of the canonical generators of Demazure modules ([J]), at least, they can not be viewed as (semi)-primitive generators of the simple modules with respect to another choice of the basis in the root system of our algebra.

We have to note that the notion of the Schubert filtration is used differently in the literature (see e.g. [P]). In this paper we will use the terminology from [Z].

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Let us briefly describe the structure of the paper. In Section 2 we collect all necessary preliminaries. In Section 3 we obtain an analogue for the Demazure formula. In Section 4 we define the Mathieu's localization and describe its basic properties. Finally, in Section 5 we construct an analogue of the Schubert filtration.

2 Notations and preliminary results

Let $\mathbb{C}$ denote the complex field; $\mathbb{Z}$ the set of integers; $\mathbb{N}$ the set of positive integers and $\mathbb{Z}_+$ the set of non-negative integers. All the notation that will be used without preliminary definition can be found in [D, Z]. For a Lie algebra $\mathfrak{A}$ by $U(\mathfrak{A})$ we will denote the universal enveloping algebra of $\mathfrak{A}$.

Consider a simple simply-laced complex finite-dimensional Lie algebra $\mathfrak{G}$. Let $\mathfrak{H}$ be a Cartan subalgebra of $\mathfrak{G}$ and Δ be the corresponding root system. Fix a basis, π, in Δ and an element, $\alpha \in \pi$. This lead to the decomposition $\Delta = \Delta_+ \cup \Delta_-$. We will denote by ρ the half-sum of all positive roots. Let W denote the Weyl group of Δ (acting in $\mathfrak{H}^*$), generated by the reflections s_β, $\beta \in \Delta$. Fix a Weyl-Chevalley basis in $\mathfrak{G}$ consisting of the elements $X_{\pm\beta}$, $\beta \in \Delta_+$ and H_β, $\beta \in \pi$. By $\mathfrak{N}_\pm$ we will denote the standard components of the triangular decomposition of $\mathfrak{G}$ with respect to the basis π. Let $\mathfrak{G}(\alpha)$ be the Lie subalgebra of $\mathfrak{G}$ generated by $X_{\pm\alpha}$ and D be the subalgebra generated by $\mathfrak{G}(\alpha)$, $\mathfrak{H}$ and $\mathfrak{N}_+$. Let $c = (H_\alpha + 1)^2 + 4X_{-\alpha}X_\alpha$ be a Casimir operator of $U(\mathfrak{G}(\alpha))$. Let Δ^α be the root subsystem of Δ generated by $\pi \setminus \{\alpha\}$. Set $K = \Delta_+ \setminus \Delta^\alpha$. Let W^α denote the Weyl group of Δ^α and ρ^α denote the half-sum of all positive roots in Δ^α. Denote by $\mathfrak{G}^\alpha$ the Lie subalgebra of $\mathfrak{G}$ corresponding to Δ^α and set $\mathfrak{N}^\alpha_\pm = \mathfrak{N}_\pm \cap \mathfrak{G}^\alpha$. We also denote by $\hat{K}$ the unital subalgebra of $U(\mathfrak{G})$ generated by all X_β, $\beta \in -K$. Recall that the Chavalley involution gives rise to a duality (exact contravariant fanctor), $*$, on the category of weight $\mathfrak{G}$-modules with finite-dimensional weight spaces. This duality preserves the character of a module. Recall ([FM2]) that a weight $\mathfrak{G}$-module is called α-stratified if both $X_{\pm\alpha}$ act injectively on it.

Consider the vector space $\Omega = \mathfrak{H}^* \oplus \mathbb{C}$. With each $(\lambda, p) \in \Omega$ there is a canonical way to associate a so-called generalized Verma module (GVM) as it was done, for example, in [FM2]. Indeed, consider a unique indecomposable $\mathfrak{G}(\alpha)$-module $N(\lambda, p)$ defined by the following conditions:

- $\lambda(H_\alpha) - 1$ is an eigenvalue of H_α on $N(\lambda, p)$;
- p^2 is the eigenvalue of c on $N(\lambda, p)$;
- all weight subspaces of $N(\lambda, p)$ are one-dimensional;
- $X_{-\alpha}$ acts bijectivly on $N(\lambda, p)$.

Clearly, $N(\lambda, p)$ can be considered as a D-module by making use of $\lambda - \rho$. The induced module

$$M(\lambda, p) = U(\mathfrak{G}) \bigotimes_{U(D)} N(\lambda, p)$$

is the GVM associated with $\mathfrak{G}$, $\mathfrak{H}$, π, α, λ and p. Clearly, $M(\lambda,p)$ has a unique simple quotient which will be denoted by $L(\lambda,p)$. There is an action of W on Ω defined as follows ([FM2]): Consider a partition of π: $\pi=\pi_1\cup\pi_2$ where $\pi_1=\{\gamma\in\pi\,|\,\alpha+\gamma\in\Delta\}$, $\pi_2=\{\gamma\in\pi\,|\,\alpha+\gamma\notin\Delta\}$. For $(\lambda,p)\in\Omega$ and $\beta\in\pi_1$ denote

$$n_\beta^\pm(\lambda,p)=\frac{1}{2}(\lambda(H_\alpha+2H_\beta)\pm p)$$

and define $(\lambda_\beta,p_\beta)\in\Omega$, where $\lambda_\beta=\lambda-n_\beta^-(\lambda,p)\beta$, $p_\beta=n_\beta^+(\lambda,p)$.

For each $\beta\in\pi$ set

$$s_\beta(\lambda,p)=\begin{cases}(\lambda,-p), & \beta=\alpha\\ (s_\beta\lambda,p), & \beta\in\pi_2\setminus\{\alpha\}\\ (\lambda_\beta,p_\beta), & \beta\in\pi_1.\end{cases}$$

Let $r\in\mathbb{C}$. Consider an affine space, $B_r=\sum_{\beta\in\pi\setminus\{\alpha\}}\mathbb{C}\beta+r\alpha$, which can be viewed as a linear space with zero (fixed point) $r\alpha$, and let $\Omega_r=B_r\times\mathbb{C}$. There exists a root system, $\Delta_{\alpha,r}$ in Ω_r, with respect to which W is the Weyl group (with the action defined above). Let $(\cdot,\cdot)_r$ denote the corresponding non-degenerated form on Ω_r and $\zeta=\zeta_{\alpha,r}:\Delta\to\Delta_{\alpha,r}$ be a natural bijection.

Let (λ,p), $(\mu,q)\in\Omega_r$ and $\beta\in\Delta_{\alpha,r}$. We will write $(\lambda,p)\overset{\beta}{\to}(\mu,q)$ if $(\mu,q)=s_\beta(\lambda,p)$ and $(\beta,(\lambda,p))_r\in\mathbb{N}$ for $\beta\neq\zeta(\alpha)$. Then $(\mu,q)\ll(\lambda,p)$ will describe the fact that there exists a sequence β_1, $\beta_2,\dots,$ β_k in $\Delta_{\alpha,r}$ such that $(\mu,q)\overset{\beta_1}{\to}s_{\beta_1}(\mu,q)\overset{\beta_2}{\to}\dots\ s_{\beta_{k-1}}\dots s_{\beta_1}(\mu,q)\overset{\beta_k}{\to}(\lambda,p)$. Denote by P^{++} the set of all $(\lambda,p)\in\Omega$ such that $w(\lambda,p)\ll(\lambda,p)$ for all $w\in W$.

For a weight $\mathfrak{G}$-module V by $\operatorname{ch}V$ we will denote its (formal) character, see [D, Section 7.5]. The main result of [FM1] is the following:

Theorem 1. *Suppose that $(\lambda,p)\in P^{++}$. Then there exists an element $a(\lambda,p)\in\mathfrak{H}^*$ such that*

$$\operatorname{ch}(L(\lambda,p))=\left(\sum_{i=-\infty}^{+\infty}e^{i\alpha}\right)\left(\prod_{\beta\in-K\setminus\{\alpha\}}(1-e^{-\beta})^{-1}\right)\times$$
$$\times\left(\sum_{w\in W^\alpha}(-1)^{l(w)}e^{w(\lambda-\rho-a(\lambda,p)+\rho^\alpha)+a(\lambda,p)}\right)\left(\sum_{w\in W^\alpha}(-1)^{l(w)}e^{w(\rho^\alpha)}\right)^{-1}.$$

3 Demazure formula for α-stratified modules

In this section we obtain an analogue for the Demazure character formula in the case of simple modules without highest weights. Suppose that $(\lambda,p)\in P^{++}$. For a positive $\beta\in\Delta^\alpha$ set

$$d_\beta=(1-e^\beta)^{-1}(1-e^\beta s_\beta).$$

Let

$$T = \left(\sum_{i=-\infty}^{+\infty} e^{i\alpha}\right)\left(\prod_{\beta\in -K\setminus\{\alpha\}} (1-e^{-\beta})^{-1}\right).$$

We recall that there is the standard length function on W (resp. W^α) with respect to π (resp. $\pi\setminus\{\alpha\}$), see [H].

Theorem 2. *There exists an element $a(\lambda,p)\in\mathfrak{H}^*$ such that for any reduced decomposition $\hat{w}_0 = s_{\beta_1}s_{\beta_2}\dots s_{\beta_k}$ of the longest element $\hat{w}_0\in W^\alpha$ the following equality holds:*

$$\operatorname{ch} L(\lambda,p) = Te^{a(\lambda,p)}\left(d_{\beta_1}d_{\beta_2}\dots d_{\beta_k}e^{\lambda-\rho-a(\lambda,p)}\right).$$

Proof. By virtue of Theorem 1, there exists $a(\lambda,p)\in\mathfrak{H}^*$ such that $\operatorname{ch} L(\lambda,p) = Te^{a(\lambda,p)}Q$, where

$$Q = \left(\sum_{w\in W(\alpha)} (-1)^{l(w)}e^{w(\lambda-\rho-a(\lambda,p)+\rho_\alpha)}\right)\left(\sum_{w\in W(\alpha)} (-1)^{l(w)}e^{w(\rho_\alpha)}\right)^{-1}.$$

On the other hand, by the Weyl formula ([D, Theorem 7.5.9]) Q can be considered as the character of a simple finite-dimensional modules over $\mathfrak{G}^\alpha$. From the Demazure character formula, [De, Théorème 2] (or [Z, Theorem 2.5.3]), we obtain that

$$Q = d_{\beta_1}d_{\beta_2}\dots d_{\beta_k}e^{\lambda-\rho-a(\lambda,p)}$$

for any reduced decomposition $\hat{w}_0 = s_{\beta_1}s_{\beta_2}\dots s_{\beta_k}$. This completes the proof. □

To construct an analogue of the Schubert filtration associated with the formula obtained in Theorem 2 we need some auxiliary notation and definitions.

4 Mathieu's localization of $U(\mathfrak{G})$

In this section we define a localization of $U(\mathfrak{G})$ ([Ma]). It allows us to reduce our problem of construction of the Schubert filtration to the problem of construction of a special filtration in certain quotient of a Verma module with integral dominant highest weight. We have to note that originally the localization of $U(\mathfrak{G})$ in [Ma] is defined in a very general situation. Since we need it only for our case of modules induced from $sl(2)$, we will define it only in this restricted case.

Consider a multiplicative subset, $\{X_{-\alpha}^n \mid n\in\mathbb{N}\}$, in $U(\mathfrak{G})$. Let $U(\alpha)$ denote the localization of $U(\mathfrak{G})$ with respect to this set, which is well-defined by [Ma, Lemma 4.2]. Let V be a weight $\mathfrak{G}$-module such that $X_{-\alpha}$ acts bijectivly on V (we will call such module normal). Then the $U(\alpha)$-module $V(\alpha) = U(\alpha)\otimes_{U(\mathfrak{G})} V$ can be naturally identified with V.

For any $x \in \mathbb{Z}$ the map $\theta_x : u \to X_{-\alpha}^{x} u X_{-\alpha}^{-x}$ is an automorphism of $U(\alpha)$. By [Ma, Lemma 4.3] this can be extended to a one-parameter family, $\{\theta_x \mid x \in \mathbb{C}\}$, of automorphisms of $U(\alpha)$, which satisfies the following condition: the map $x \mapsto \theta_x(u)$ is polynomial in x for any $u \in U(\alpha)$. For a $U(\alpha)$-module W and $x \in \mathbb{C}$ we will denote by $\theta_x(W)$ the shift of W by θ_x, i.e. a $U(\alpha)$-module which is equal to W as a vector space and $u \cdot w = \theta(u)w$ for all $u \in U(\alpha)$ and $w \in W$. Clearly, one can consider any $U(\alpha)$-module as a $U(\mathfrak{G})$-module by restriction.

We emphasize that, directly from the definition of θ_x, it follows that $\theta_x(c) = c$ and for any weight vector v of weight λ, $\theta_x(v)$ is a weight vector of weight $\lambda + x\alpha$.

5 Schubert filtration for α-stratified modules

The aim of this section is to prove the following theorem, which presents a natural generalization of the Schubert filtration of a simple finite-dimensional module.

Theorem 3. *Let (λ, p) be an element in P^{++}. There is an element $a(\lambda, p) \in \mathfrak{H}^*$ such that for any reduced decomposition $\hat{w}_0 = s_{\beta_1} s_{\beta_2} \dots s_{\beta_k}$ there exists a (canonical) filtration*

$$L(\lambda, p) = L_1 \supset L_2 \supset \cdots \supset L_k \supset L_{k+1} = 0$$

of $L(\lambda, p)$ (viewed as a D-module) by D-modules L_j, $j = 1, 2, \dots, k$, such that

$$\mathrm{ch}\, L_j = T e^{a(\lambda,p)} \left(d_{\beta_j} \dots d_{\beta_k} e^{\lambda - \rho - a(\lambda,p)} \right).$$

Proof. To simplify our notation we set $L = L(\lambda, p)$.

Step 1. First we reduce our problem to highest weight modules. According to [Ma, Proposition 5.7], there exists $x \in \mathbb{C}$ such that the element X_α has a non-trivial kernel on $\theta_x(L)$. In our case this can also be shown by direct computation with $sl(2)$. Indeed, consider the generating $\mathfrak{G}(\alpha)$-submodule $1 \otimes N(\lambda, p)$ of L. From the weight space decomposition it is clear that this submodule is a $\mathfrak{G}(\alpha)$-direct summand of L. From the definitions of θ_x and $N(\lambda, p)$ one has that, as a $\mathfrak{G}(\alpha)$-module, $\theta_x(N(\lambda, p)) \simeq N(\lambda', p)$, where $\lambda(H_\alpha) + 2x = \lambda'(H_\alpha)$. Now we can choose x such that $(\lambda'(H_\alpha)+1)^2 = p^2$ which guarantees the existence of a non-trivial element, annihilated by X_α. Hence, the module $L' = \theta_x(L)$ is not α-stratified. As $1 \otimes N(\lambda, p)$ generates L, its twist generates L'. Denote by $L(1)$ the submodule of L' consisting of all locally X_α-finite elements. Then $L(2) = L'/L(1)$ is a highest weight module with respect to the base $s_\alpha(\pi)$. Now both $L(1)$ and $L(2)$ are generated by their intersections with $N(\lambda', p)$, in particular, the highest weight of $L(1)$ and the $s_\alpha(\pi)$-highest weight of $L(2)$ are one-dimensional.

Denote by μ the highest weight of $L(1)$. From the arguments above (or directly from the properties of Mathieu's localization) we have that $\mu = \lambda - \rho + y\alpha$ for some $y \in \mathbb{C}$. Recall that $(\lambda, p) \in P^{++}$. For any simple root $\beta \neq \alpha$ from $(\lambda, p) \stackrel{\beta}{\to} s_\beta(\lambda, p)$ by direct calculation we have $s_\beta(\mu + \rho) \leqslant \mu + \rho$ (with respect to the standard partial order on $\mathfrak{H}^*$). If $\alpha + \beta$ is a root we also have $s_\alpha s_\beta s_\alpha(\lambda, p) \ll (\lambda, p)$ from which one derives $p \in \mathbb{Z}$. Since $L(1)$ is the maximal $\mathfrak{G}$-submodule of L' on which X_α acts locally nilpotent, we have $s_\alpha(\mu+\rho) \leqslant \mu+\rho$ and $\mu(H_\alpha) = |p| - 1$. Altogether, we get that μ is in fact integral dominant and a unique simple quotient of $L(1)$ is finite-dimensional.

Step 2. Consider a $\mathfrak{G}^\alpha$-module $M = U(\mathfrak{G}^\alpha)L(1)_\mu$. This is a highest weight $\mathfrak{G}^\alpha$-module. The support of any non-trivial $\mathfrak{G}^\alpha$-submodule of M does not intersects the highest weight of M and thus this submodule generates in $L(1)$ a non-trivial $\mathfrak{G}$-submodule, whose support does not intersect the intersection of $L(1)$ with $N(\lambda', p)$. Inducing to $U(\alpha)$, this gives a non-trivial $\mathfrak{G}$-submodule in L' on which $X_{-\alpha}$ acts bijectively. And after the return twist with θ_{-x} we get that L is not simple, which contradicts our assumptions. Hence, M does not contain any non-trivial $\mathfrak{G}^\alpha$-submodule, therefore M is a simple $\mathfrak{G}^\alpha$-module. As μ is integral doiminant we also get that M is finite-dimensional.

Consider a reduced decomposition $\hat{w}_0 = s_{\beta_1}s_{\beta_2}\dots s_{\beta_k}$. According to the classical Demazure character formula and the classical Schubert filtration one can consider a Schubert filtration

$$M = M_1 \supset M_2 \supset \dots \supset M_k \supset M_{k+1} = 0$$

of M by $\mathfrak{N}_+^\alpha$-module, corresponding to the above decomposition of $\hat{w}_0$, with $\mathrm{ch}\, M_i$ given by the Demazure formula. Let v_j denote a (canonical) generator of M_j for $j \in \{1, 2, \dots, k\}$.

Step 3. By [FM1, Theorem 4], the maximal submodule of $M(\lambda, p)$ is generated by the images of all $M(s_\beta(\lambda, p))$ in $M(\lambda, p)$, where β runs through the set of all simple roots different form α. Lifting this to $L(1)$ we get that $L(1)$ is the quotient of the Verma module $M(\mu+\rho)$ over the submodule generated by the images of all $M(s_\beta(\mu+\rho))$ in $M(\mu+\rho)$, where $\beta \neq \alpha$ is simple.

By PBW Theorem, $U(\mathfrak{N}_-)$ is free over $\hat{K}$ with the basis $U(\mathfrak{N}_-^\alpha)$. We can identify each Verma module with $U(\mathfrak{N}_-)$ as an $\mathfrak{N}_-$-module. We have that $M(\mu+\rho)$ is free over $\hat{K}$ with the basis $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu$. Denote by $\hat{M}$ the submodule of $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu$ generated by the intersection of $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu$ with all images of $M(s_\beta(\mu+\rho))$ in $M(\mu+\rho)$, where $\beta \neq \alpha$ is simple. We have $L(1) \simeq M(\mu+\rho)/U(\mathfrak{G})\hat{M}$ and $U(\mathfrak{G})\hat{M} = U(\mathfrak{N}_-)\hat{M} = \hat{K}\hat{M}$. Since $\hat{M}$ is a subset of $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu$ and the last one is a $\hat{K}$-free basis of $M(\mu+\rho)$, we get that $L(1)$ is $\hat{K}$-free with a basis $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu/\hat{M} \simeq M$ (here we mean that any $\mathbb{C}$-basis of $U(\mathfrak{N}_-^\alpha)M(\mu+\rho)_\mu$, resp. $L(1)$ is a $\hat{K}$-free basis).

Step 4. Let N be arbitrary $\mathfrak{N}_-^\alpha$-submodule of M. Then $\hat{K}N$ is a D^*-module (here and on $*$ is with respect to the Chevalley involution), moreover $\mathrm{ch}\, \hat{K}N = \mathrm{ch}\, \hat{K} \times \mathrm{ch}\, N$. Indeed, the character formula follows from the free action of $\hat{K}$. Clearly, $\hat{K}N$ is a $\hat{K}$-module and an $\mathfrak{H}$-module. As $[\mathfrak{N}_-^\alpha, \hat{K}] \subset \hat{K}$, $\hat{K}N$ is a $\mathfrak{N}_-^\alpha$-module. So, it is enough to show that $\hat{K}N$ is closed under the action of X_α. This now follows from the fact that $[X_\alpha, \hat{K}] \subset U(\mathfrak{N}_- \oplus \mathfrak{H})$ and $X_\alpha N = X_\alpha M = 0$.

Step 5. For each $1 \leqslant i \leqslant k$ there exists a $\mathfrak{N}_-^\alpha$-submodule, N_i, of M such that $\mathrm{ch}\, M/N_i = \mathrm{ch}\, M_i$. Moreover, we can choose N_i such that $N_i \subset N_{i+1}$. Indeed, as M_i is a $\mathfrak{N}_+^\alpha$-submodule of $M \simeq M^*$, we can apply $*$ and obtain a $\mathfrak{N}_-^\alpha = (\mathfrak{N}_+^\alpha)^*$-submodule of $M \simeq M^*$, whose quotient is isomorphic to M_i^*. As $*$ preserves the character this is exactly

N_i we need. The statment about inclusions for N_i's follows from the opposite inclusions of M_i's and contravariantness of $*$.

Step 6. Set $I_i = \hat{K}N_i$. By Steps 4 and 5 we have $\operatorname{ch} I_i = \operatorname{ch}\hat{K} \times (\operatorname{ch} M - \operatorname{ch} M_i)$. Clearly, $X_{-\alpha}$ acts injectively on I_i. Inducing I_i up to $U(\alpha)$ and shifting by θ_x we obtain a filtration, $\hat{L}_i$, of L by D^*-modules. Moreover, as $X_{-\alpha}$ acts injectively on $\hat{K}$, $\operatorname{ch}\hat{K}$ changes to T during the induction process, therefore we obtain that $\operatorname{ch}\hat{L}_i = T \times (\operatorname{ch} M - \operatorname{ch} M_i)$. Note that $N(\lambda, p)^* \simeq N(\lambda, p)$ as $N(\lambda, p)$ is α-stratified and hence $L^* \simeq L$ since $*$ preserves the character of the module and L is completely determined by (λ, p). Applying the duality one more time we get that there exists a filtration of $L \simeq L^*$ by $U(D)$-modules L_i such that $\operatorname{ch} L_i = \operatorname{ch} L - \operatorname{ch}\hat{L}_i$. Now the desired result for this filtration L_i follows from the fact that $\operatorname{ch} L = T \times \operatorname{ch} M$ (Theorem 2). $\square$

We remark that, with the same arguments, the statements of Theorem 2 and Theorem 3 are true for $L(\hat{w}_0 w_0(\lambda, p))$, where $(\lambda, p) \in P^{++}$ and w_0 is the longest element of W.

6 Acknowledgments

The second author is partially supported by CRDF grant. The paper was completed during the stay of the second author in Bielefeld University as an Alexander von Humboldt fellow. Financial support of CRDF, Humboldt Foundation and hospitality of Bielefeld University are gratefully acknowledged. We also thank the referee for his comments that led to the improvements in the paper.

References

[A] H.Andersen, Schubert varieties and Demazure character formula. Invent. Math. 5 (1985), 611-618.

[De] M.Demazure, Désingularisation des variétés de Schubert généralisées. Ann. Ec. Norm. Sup. 7 (1974), 53-88.

[D] J.Dixmier, Enveloping algebras. Revised reprint of the 1977 translation. Graduate Studies in Mathematics, 11. American Mathematical Society, Providence, RI, 1996.

[J] A.Joseph, Quantum Groups and Their Primitive Ideals, Springer-Verlag, New-York, 1994.

[FM1] V.Futorny, V.Mazorchuk, BGG-resolution for α-stratified modules over simply-laced finite-dimensional Lie algebras. J. Math. Kyoto Univ. 38 (1998), 229-240.

[FM2] V.Futorny, V.Mazorchuk, Structure of α-Stratified Modules for Finite-Dimensional Lie Algebras, I, Journal of Algebra, 183 (1996), 456-482.

[H] J. Humphreys, Reflection groups and Coxeter groups, Cambridge University Press, Cambridge, 1990.

[Ma] O.Mathieu, Classification of irreducible weight modules, Preprint, Strasbourg University, 1997.

[M] V.Mazorchuk, On the structure of an α-stratified generalized Verma Module over Lie algebra $sl(n,\mathbb{C})$, Manuscripta Math., 88 (1995), 59-72.

[MO] V.Mazorchuk and S.Ovsienko, Submodule structure of Generalized Verma Modules induced from generic Gelfand-Zetlin modules, Algebr. Represent. Theory 1 (1998), 3-26.

[P] P.Polo, Un critère d'existence d'une filtration de Schubert. (French) [A criterion for the existence of a Schubert filtration], C. R. Acad. Sci. Paris Sér. I Math. 307 (1988), 791-794.

[Z] D.Zelobenko, Representations of reductive Lie algebras, Moskow, "Nauka", 1994. (in Russian).

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